



JEE(Main) : 2021

(Questions with Answer)

Date : 24th February 2021 (Shift-2) | Time : 3.00 p.m. to 6.00 p.m|

Duration : 3 Hours | Max. Marks : 300

(Memory Based Question)

PHYSICS

CHEMISTRY

MATHEMATICS

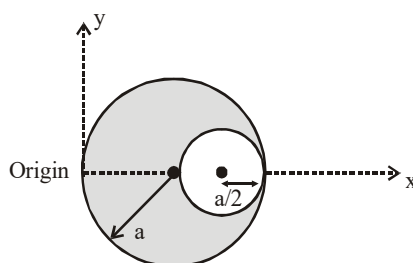
PHYSICS

1. Which of the following is correct for zener diode
- (1) Lightly doped, depletion layer thickness is less
 - (2) Heavily doped, depletion layer thickness is less
 - (3) Lightly doped, depletion layer thickness is more
 - (4) Heavily doped, depletion layer thickness is more.

Ans. (4)

Sol. It is heavily doped and work in reverse biased so depletion layer thickness is more.

2. Disc of radius $\frac{a}{2}$ is cut from disc of radius a . Find x-co-ordinate of COM from origin.



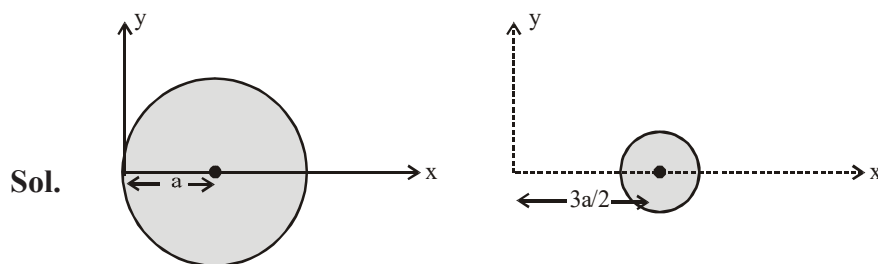
(1) $\frac{5a}{6}$

(2) $\frac{a}{6}$

(3) $\frac{a}{3}$

(4) $\frac{a}{2}$

Ans. (1)



Sol.

$$x_{\text{com}} = \frac{(\sigma \times \pi a^2 \times a) - \sigma \pi \frac{a^2}{4} \times 3 \frac{a}{2}}{\sigma \pi a^2 - \sigma \frac{\pi a^2}{4}}$$

$$x_{\text{com}} = \frac{a - \frac{3a}{4}}{1 - \frac{1}{4}} = \frac{\frac{5a}{4}}{\frac{3}{4}} = \frac{5a}{6}$$

3. A body is projected from origin with a velocity v_0 along +ve x-axis on which a force $-\alpha x^2$ is acting on it. Find the maximum distance from origin up to which the body can go. (x is the position of particle).

(1) $\left(\frac{3mv_0^2}{2\alpha}\right)^{\frac{1}{3}}$ (2) $\left(\frac{3mv_0^2}{2\alpha}\right)^{\frac{1}{2}}$ (3) $\left(\frac{2mv_0^2}{3\alpha}\right)^{\frac{1}{3}}$ (4) $\left(\frac{2mv_0^2}{3\alpha}\right)^{\frac{1}{2}}$

Ans. (1)

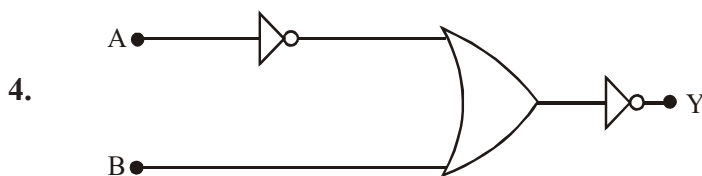
Sol. $F = -\alpha x^2 = ma$

$$a = \frac{-\alpha x^2}{m} = v \frac{dv}{dx}$$

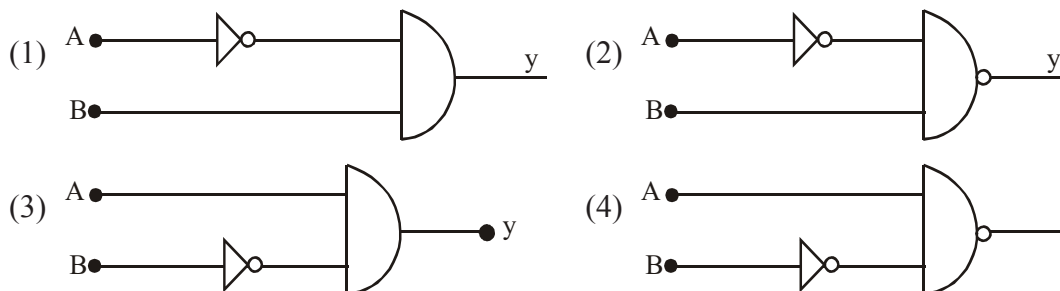
$$\int_{v_0}^0 v dv = \int_0^x -\frac{\alpha x^2}{m} dx$$

$$-\frac{v_0^2}{2} = -\frac{\alpha x^3}{3m}$$

$$x = \left(\frac{3mv_0^2}{2\alpha}\right)^{\frac{1}{3}}$$



This diagram is equivalent to



Ans. (3)

Sol. $y = \overline{\overline{A} + B} = A\overline{B}$

which is equivalent to (3) option

5. Weight at pole is 49 Newton then then weight at equator?

(1) 48.83 N (2) 49.17 N (3) 49.83 N (4) 49 N

Ans. (1)

Sol. $mg = 49$
 $m(g - \omega^2 R)$ will be less than mg .

6. If the de-Broglie wavelengths of α -particle and proton are equal then the ratio of their velocities are

- (1) $\frac{1}{4}$ (2) $\frac{4}{1}$ (3) $\frac{1}{2}$ (4) 1

Ans. (1)

Sol. $\frac{h}{m_\alpha v_\alpha} = \frac{h}{m_p v_p}$
 $\Rightarrow \frac{u_\alpha}{u_p} = \frac{m_p}{m_\alpha} = \frac{1}{4}$

7. A light source emitting light and if we change light from violet to red then

- (1) Fringes become dark
 (2) The separation between two consecutive fringes decreases
 (3) Intensity of central maxima will increase
 (4) Intensity of central maxima will decrease

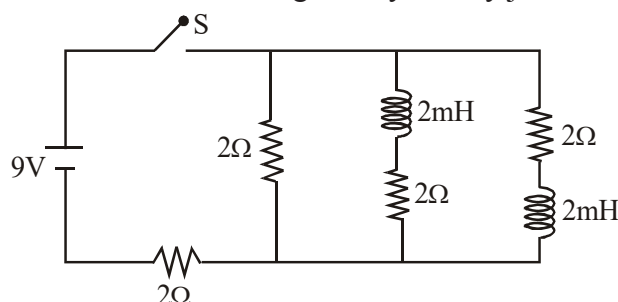
Ans. (1)

Sol. $\beta = \frac{\lambda D}{d}$, $\lambda_V < \lambda_R$

$$\beta_V < \beta_R$$

There is no change in intensity of bright and dark fringes.

8. Switch S is closed at $t = 0$. Find the current given by battery just after closing the switch:

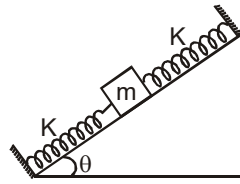


- (1) 2.25 A (2) 3.36 A (3) zero (4) 4.5 A

Ans. (1)

Sol. $i = \frac{9}{4} \text{ A} = 2.25 \text{ A}$

9. Find frequency of oscillation of spring block system shown.



- (1) $\frac{1}{2\pi} \sqrt{\frac{2K}{m}}$ (2) $\frac{1}{2\pi} \sqrt{\frac{4K}{m}}$ (3) $\frac{1}{2\pi} \sqrt{\frac{6K}{m}}$ (4) $\frac{1}{2\pi} \sqrt{\frac{K}{2m}}$

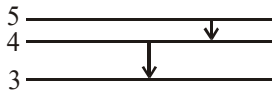
Ans. (1)

Sol. $T = 2\pi \sqrt{\frac{m}{2K}} \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{2K}{m}}$

10. For which of the following transitions in Hydrogen-like atom of the frequency of emitted photon will be maximum

- (1) $n = 2$ to $n = 1$ (2) $n = 4$ to $n = 3$ (3) $n = 5$ to $n = 4$ (4) $n = 3$ to $n = 2$

Ans. (1)



Sol.

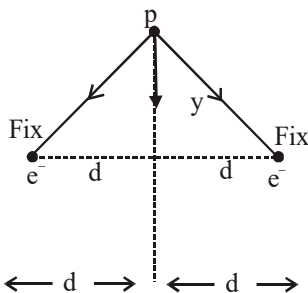


f is more for transitions from $n = 2$ to $n = 1$

11. Two electron are fix, at separation $2d$. At middle of these two charges a proton is free to oscillate. Mass of proton is m_p and charge of proton is e .
Now if proton is slightly displaced perpendicular to line joining of charge then find it's angular frequency of oscillation.

- (1) $\sqrt{\frac{e^2}{2\pi \epsilon_0 m_p d^3}}$ (2) $\sqrt{\frac{2e^2}{\pi \epsilon_0 d^2 m_p}}$ (3) $\sqrt{\frac{2e^2 \epsilon_0 m_p}{\pi d^2}}$ (4) $\sqrt{\frac{2 \epsilon_0 e^2 m_p}{\pi d^2}}$

Ans. (1)



Sol.

Restoring force on proton

$$F_r = \frac{2Ke^2y}{(d^2 + y^2)^{3/2}} \quad y \ll d$$

$$F_r = \frac{2Ke^2y}{d^3} = \frac{e^2y}{2\pi\epsilon_0 d^3} = ky$$

$$k = \frac{e^2}{2\pi\epsilon_0 d^3}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{e^2}{2\pi\epsilon_0 md^3}}$$

12. If soft iron (ferromagnetic substance) is placed in external uniform magnetic field.

- (1) Size of domain will change and orientation also change.
- (2) Size of domain will increase
- (3) Size of domain will decrease
- (4) Does not depend on external magnetic field.

Ans. (1)

Sol. Theoretical

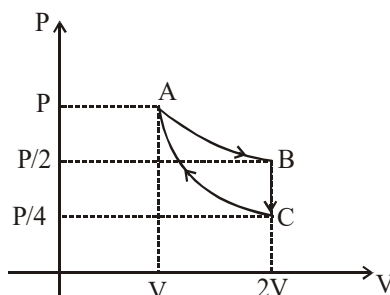
13. Which of the following represent travelling wave?

- (1) $y = A \sin (15 t - 2x)$
- (2) $y = A \cos (15 t) \sin 2x$
- (3) $y = A \sin (15 t) \cos 2x$
- (4) $y = A e^{-x} \cos (15 t - 2x)$

Ans. (1)

Sol. travelling wave function is $f(t \pm x/v)$

14. P-V diagram is shown. AB is Isothermal process and during AB temperature is T. Gas is 1 mole, BC is isochoric and CA is adiabatic process. Coefficient of adiabatic exponent is γ . Find total work done by gas.



- (1) $RT \left[\ln 2 - \frac{1}{2(\gamma-1)} \right]$ (2) $RT \ln 2$
 (3) $\frac{RT}{2(\gamma-1)}$ (4) zero

Ans. (1)

Sol. $W_{AB} = nRT \ln 2 = RT \ln 2$

$$W_{BC} = 0$$

$$W_{CA} = \frac{PV - \frac{P}{4} \times 2V}{1-\gamma} = \frac{PV}{2(1-\gamma)}$$

$$W_{ABCA} = RT \ln 2 + \frac{RT}{2(1-\gamma)}$$

$$RT \left[\ln 2 - \frac{1}{2(\gamma-1)} \right]$$

15. Which of the following are correctly matches.

- | | |
|-------------------|---|
| (A) x-ray | (i) Inter electron transition |
| (B) γ -ray | (ii) Vibration of atom and molecule |
| (C) Radio-wave | (iii) radioactivity |
| (D) Micro-wave | (iv) LASER |
| | (v) Magnetron |
| | (vi) Rapid acceleration and deceleration of electron in aerials |

(1) (A) $\rightarrow$ (i), (B) $\rightarrow$ (iii), (C) $\rightarrow$ (vi), (D) $\rightarrow$ (v)

(2) (A) $\rightarrow$ (ii), (B) $\rightarrow$ (iii), (C) $\rightarrow$ (vi), (D) $\rightarrow$ (i)

(3) (A) $\rightarrow$ (i), (B) $\rightarrow$ (iii), (C) $\rightarrow$ (ii), (D) $\rightarrow$ (vi)

(4) (A) $\rightarrow$ (v), (B) $\rightarrow$ (iii), (C) $\rightarrow$ (vi), (D) $\rightarrow$ (ii)

Ans. (1)

Sol. Theoretical information based

16. Time period of simple pendulum is given by $T = 2\pi \sqrt{\frac{\ell}{g}}$.

Find % error in g when T is 1.95 sec and $\ell = 1$ meter.

Given least count in $\ell = 1\text{mm}$ and least count in $T = 10$ milli second, find percentage error in g

- (1) 2.2% (2) 1.33% (3) 1.12% (4) 1.03%

Ans. (3)

Sol. $T^2 = 4\pi^2 \left[\frac{\ell}{g} \right]$

$$g = 4\pi^2 \left[\frac{\ell}{T^2} \right]$$

$$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + \frac{2\Delta T}{T}$$

$$= \left[\frac{1\text{mm}}{1\text{m}} + \frac{2(10 \times 10^{-3})}{1.95} \right] \times 100$$

$$= 1.12 \%$$

17. In x-ray tube experiment accelerating voltage is given by 1.24 M volt. Find cut of wavelength.

- (1) 10^{-3} nm (2) 10^{-2} nm (3) 10^{-4} nm (4) 10^{-5} nm

Ans. (1)

Sol. $\lambda_{\min} = \frac{\lambda c}{eV} = \frac{1240\text{nm} - eV}{1.24 \times 10^6}$

$$\lambda_{\min} = 10^{-3} \text{ nm}$$

18. Read the given statement

S-1 A transistor can be made by using two P-N diode connected in series.

S-2 β is ratio of $\frac{I_C}{I_B}$

- (1) S-1 is true, S-2 is false
 (2) S-1 is false, S-2 is true
 (3) S-1 is true, S-2 is true
 (4) S-1 is false, S-2 is false

Ans. (2)

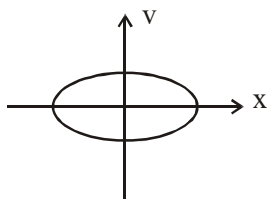
Sol. Theoretically

19. Which of the following curve represents relation between velocity with displacement of particle performing simple harmonic motion

- (1) ellipse (2) circle (3) hyperbola (4) straight line

Ans. (1)

Sol. $v = \omega \sqrt{A^2 - x^2}$



20. Which of the following is most appropriate regarding Bohr model.

(A) speed $\propto \frac{Z}{n}$

(B) frequency of oscillation $\propto \frac{Z^3}{n^3}$

(C) coulombic force of attraction $\propto \frac{Z^3}{n^4}$

(D) Kinetic Energy $\propto \frac{Z^2}{n^2}$

(1) A,C,D

(2) A,B,D

(3) B,D

(4) A,B

Ans. (1)

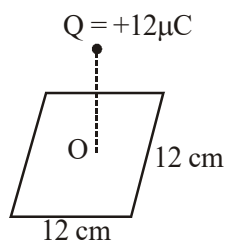
Sol. Speed $\propto \frac{Z}{n}$

frequency of oscillation $\propto \frac{Z^2}{n^3}$

coulombic force of attraction $\propto \frac{Z^3}{n^4}$

Kinetic Energy $\propto \frac{Z^2}{n^2}$

21. A point charge $Q = 12 \mu\text{C}$ at distance 6 cm from centre of square of side 12cm as shown in figure. Electric flux through square is $x \times 10^3$ S.I. units. Find x.



Ans. 226

Sol. using Gauss law it is a part of cube of side 12 cm and charge at centre so $\Phi = \frac{Q}{6\epsilon_0} = \frac{12\mu\text{C}}{6\epsilon_0}$

$$x \times 10^3 = 2 \times 4\pi \times 9 \times 10^9 \times 10^{-6}$$

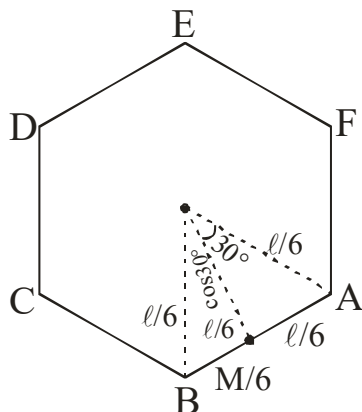
$$\Phi = 72 \pi \times 10^3 \text{ SI units}$$

$$x = 226$$

22. A rod of length 2.4 m and mass 6 kg is bent to form a regular hexagon. Find the moment of inertia (kgm^2) of the hexagon about the axis perpendicular to the plane of hexagon passing through its centre.

Ans. 0.8

Sol.



$$I_{AB} = \left[\frac{\frac{M}{6} \left(\frac{\ell}{6} \right)^2}{12} + \frac{M}{6} \left(\frac{\ell}{6} \frac{\sqrt{3}}{2} \right)^2 \right]$$

$$I_{\text{hexagon}} = 6I_{AB} = M \left[\frac{\ell^2}{12 \times 36} + \frac{\ell^2}{36} \times \frac{3}{4} \right]$$

$$= \frac{6}{100} \left[\frac{24 \times 24}{12 \times 36} + \frac{24 \times 24}{36} \times \frac{3}{4} \right]$$

$$= \frac{1}{100} [80] = 0.8 \text{ kgm}^2$$

23. Two objects of mass ratio 1 : 2 have ratio of kinetic energy A : 1. If their momentum are same find A?

Ans. 2

Sol. $\frac{M_1}{M_2} = \frac{1}{2}$

$$M_1 V_1 = M_2 V_2 = P$$

$$K_1 = \frac{P^2}{2M_1} \quad K_2 = \frac{P^2}{2M_2}$$

$$\frac{K_1}{K_2} = \frac{M_2}{M_1} = \frac{2}{1}$$

$$= \frac{A}{1} = \frac{2}{1} = 2$$

24. Two cars A and B both are moving towards each other with a speed of 7.2 km/hr. Source A produces a sound of frequency 676 Hz. Find the beat frequency (Hz) heard by the person sitting in car A. (speed of sound is 340 m/sec in air)

Ans. 16

Sol. frq. observed by A = $f_A = 676$ Hz

$$\text{frq. observed by B} = f_B = \left(\frac{v+u}{v-u} \right) f$$

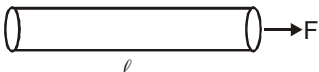
frq. observed by A after reflection from B is

$$f'_A = \left(\frac{v+u}{v-u} \right) f_B = \left(\frac{v+u}{v-u} \right)^2 f$$

$$\begin{aligned} \text{Beats frq. : } f_b &= f'_A - f_A = \left[\left(\frac{v+u}{v-u} \right) - 1 \right] f \\ &= \left[\frac{v^2 + u^2 + 2uv - v^2 - u^2 + 2uv}{(v-u)^2} \right] f \\ &= \frac{4uv}{(v-u)} \times f \quad v \gg u \\ &= \left(\frac{4uv}{v^2} \right) f = \frac{4u}{v} \times f = 4 \times \frac{2}{340} \times 676 \text{ Hz} = 16 \text{ Hz} \end{aligned}$$

25. On applying force to a rod along its length, the rod gets elongated by 0.04m. If the length of rod is doubled and diameter is also doubled and same force is applied along the length and it is found that the rod gets elongated by $x \times 10^{-2}$ m. Find the value of x.

Ans. 2

Sol. 

$$\frac{F}{A} = y \frac{\Delta L}{L}$$

$$\frac{F}{A} = y \times \frac{0.04}{L} \quad \dots(i)$$

$$\frac{F}{4A} = y \times \frac{\Delta L}{2L} \quad \dots(ii)$$

$$4 = \frac{0.04 \times 2}{\Delta L}$$

$$\Delta L = 2 \times 10^{-2}$$

$$x = 2$$

26. There is a cylindrical wire radius 0.5 mm and conductance $5 \times 10^7 \Omega \text{m}^{-1}$, If the potential gradient along the length of cylindrical wire is 10 mV/m and the current flowing through cylindrical wire is $x^3 \times \pi$ mA. Find the value of x.

Ans. 5

Sol. $J = \sigma E$

$$= 5 \times 10^7 \times 10 \times 10^{-3}$$

$$= 50 \times 10^4 \text{ A/m}^2$$

$$I = J\pi R^2$$

$$= 50 \times 10^4 \times \pi (0.5 \times 10^{-3})^2$$

$$= 50 \times 10^4 \times \pi \times 0.25 \times 10^{-6}$$

$$= 125 \times 10^{-3} \pi$$

$$x = 5$$

- 27.** In series LCR circuit at resonance power consumption in circuit is 16W if voltage source of frequency 10^5 rad/s is 120 V then find resistance (Ω) of circuit.

Ans. 800

Sol. $\frac{(120)^2}{R} = 16$

$$R = \frac{14400}{16} = 800 \Omega$$

- 28.** A gas is at rms speed 200 m/sec at 27°C and 1 atm pressure. If it's rms speed is $\frac{x}{\sqrt{3}}$ m/s when temp is 127°C and 2 atm pressure, find value of x

Ans. 400

Sol. $V_{\text{rms}} = \sqrt{\frac{3RT}{M_0}}$

$$200 = \sqrt{\frac{3R \times 300}{M_0}}$$

$$\frac{x}{\sqrt{3}} = \sqrt{\frac{3R \times 400}{M_0}}$$

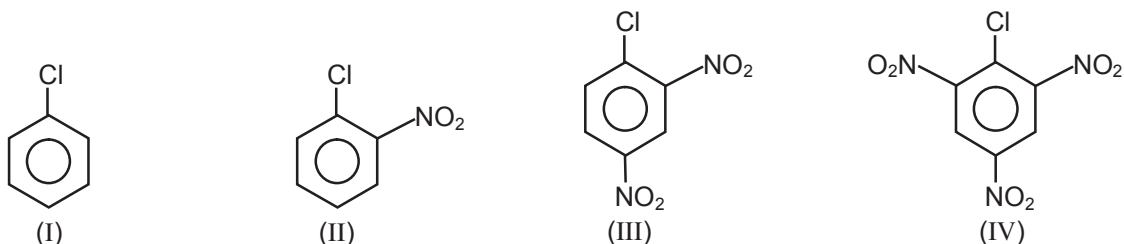
$$\frac{200}{\frac{x}{\sqrt{3}}} = \sqrt{\frac{3}{4}}$$

$$\frac{200\sqrt{3}}{x} = \frac{\sqrt{3}}{2}$$

$$X = 400 \text{ m/s}$$

CHEMISTRY

1. Compare the rate of aromatic nucleophilic substitution reaction of the following compounds



- (1) IV > II > III > I (2) III > II > I > IV (3) I > II > III > IV (4) IV > III > II > I

Ans. (4)

Sol. Rate of aromatic nucleophilic substitution reaction depends upon type of halogens and electronic effect of the group present on the ring. Electron withdrawing groups ($-I$, $-M$) increases rate of reaction increases.

2. What is S in Buna-S ?

- (1) Sulphure (2) Styrene (3) Rubber (4) Strength

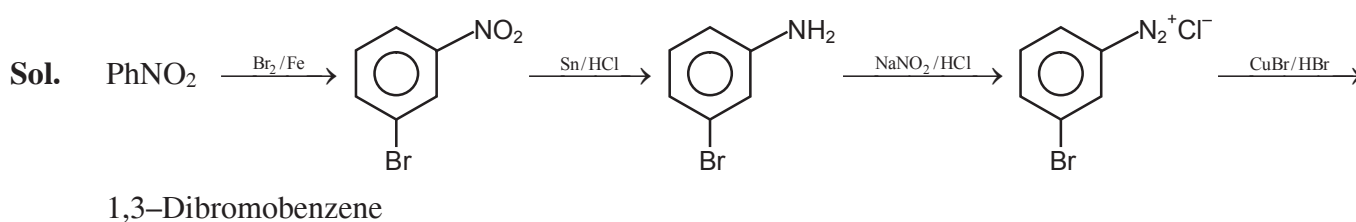
Ans. (2)

Sol. Buna-S is the co-polymer of buta-1,3-diene and styrene

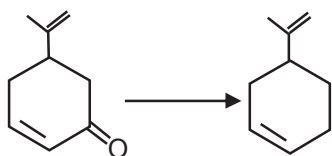
3. Which of the following set of the reagent is used to convert nitrobenzene to 1,3-Dibromobenzene?

- (1) $\text{Br}_2/\text{Fe} \longrightarrow \text{Sn}/\text{HCl} \longrightarrow \text{NaNO}_2/\text{HCl} \longrightarrow \text{CuBr}/\text{HBr}$
 (2) $\text{Br}_2/\text{Fe} \longrightarrow \text{NaNO}_2/\text{HCl} \longrightarrow \text{Sn}/\text{HCl}$
 (3) $\text{Br}_2/\text{Fe} \longrightarrow \text{NaNO}_2/\text{HCl} \longrightarrow \text{Sn}/\text{HCl} \longrightarrow \text{HBr}/\text{CuBr}$
 (4) $\text{Br}_2/\text{Fe} \longrightarrow \text{HBr}/\text{CuBr} \longrightarrow \text{NaNO}_2/\text{HCl} \longrightarrow \text{Sn}/\text{HCl}$

Ans. (1)

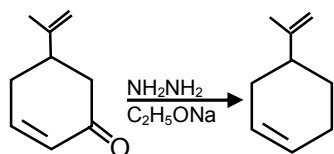


4. The reagent used to convert the following is ?



- (1) $\text{NH}_2\text{-NH}_2/\text{C}_2\text{H}_5\text{O}^-\text{Na}^+$ (2) Red P/ Cl_2
 (3) Ni/H_2 (4) NaBH_4

Ans. (1)

Sol.

it is wolff-Kishner reduction of carbonyl compounds.

5. Match the following**Column – I**

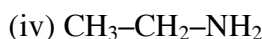
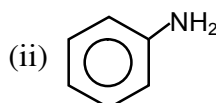
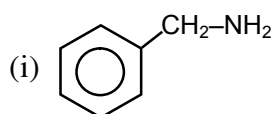
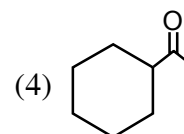
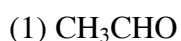
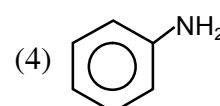
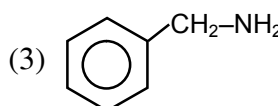
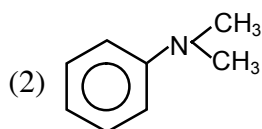
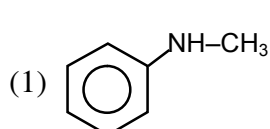
- (a) Valium
 (b) Morphine
 (c) Norethindrone
 (d) Vitamin B-12

Column – II

- (p) Pernicious anaemia
 (q) Analgesic
 (r) Tranquilizer
 (s) Antifertility

Ans. a → r ; b → q ; c → s ; d → p**6.** **Statement – I :** BOD is the parameter that can be helpful for survival of aquatic life.**Statement – II :** Optimum value of BOD is 6.5 ppm.

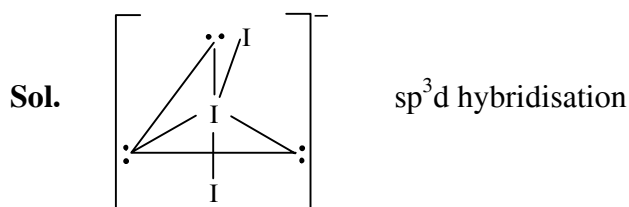
- (1) Statement I is true ,Statement II is false
 (2) Statement I is false ,Statement II is true
 (3) Statement I , II both are true
 (4) Statement I , II both are false

Ans. (1)**7.** How many of the following amines can be prepared by Gabriel phthalimide synthesis ?**Ans.** (3)**Sol.** Only aliphatic amines can be prepared by Gabriel phthalimide synthesis.**8.** Which of the following compound cannot be prepared by the reaction of alkyne with HgSO₄/dil.H₂SO₄?**Ans.** (2)**9.** Diazonium salt of which of the following will give coloured dye on reaction with β-Naphthol in NaOH**Ans.** (4)**Sol.** Only aromatic Primary amines will gives Dye test.

10. The correct bond angle & shape of I_3^- is

- (1) Linear & 180° (2) Trigonal pyramidal & 120°
 (3) V-shape & 120° (4) T-shape & $109^\circ 28'$

Ans. (1)



Linear shape $\angle \text{I-I-I} = 180^\circ$

11. Correct statements

- (a) $\text{K.E.} \propto \frac{Z^2}{n^2}$
 (b) $(nv) \propto Z^2$
 (c) $\text{Frequency} \propto \frac{Z^3}{n^3}$
 (d) $\text{Electrostatic force} \propto \frac{Z^3}{n^4}$

- (1) a & d are correct (2) a & b are correct
 (3) b & c are correct (4) b & d are correct

Ans. (1)

12. Which of the following is incorrect?

- (1) Cr_2O_3 is Amphoteric (2) RuO_4 is oxidising agent
 (3) VOSO_4 is reducing agent (4) Ruby appears due to presence of Co^{3+}

Ans. (4)

13. Which of the following order of melting point is correct

- (1) $\text{LiF} > \text{LiCl}$, $\text{NaCl} > \text{MgO}$ (2) $\text{LiF} < \text{LiCl}$, $\text{NaCl} > \text{MgO}$
 (3) $\text{LiF} > \text{LiCl}$, $\text{NaCl} < \text{MgO}$ (4) $\text{LiF} < \text{LiCl}$, $\text{NaCl} > \text{MgO}$

Ans. (3)

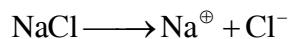
Sol. Lattice energy $\propto \left| Z^+ \right| \left| Z^- \right|$

$$\propto \frac{1}{r^+ + r^-}$$

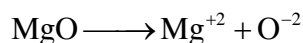
LiF LiCl

Size $\text{F}^- < \text{Cl}^-$ [charge are same]

Lattice energy $\boxed{\text{LiF} > \text{LiCl}}$



$$|Z^{+}||Z^{-}| = 1 \times |1| = 1$$

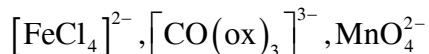


$$|2||-2| = 4$$

Lattice energy

$$\boxed{\text{MgO} > \text{NaCl}}$$

Charge dominate over size

14. Spin only magnetic moment of the following complexes

(1) 4.9, 0, 1.76 BM

(2) 5.9, 1.73 BM

(3) 1.73, 2.83, 0 BM

(4) 2.83, 6.9, 0 BM

Ans. (1)

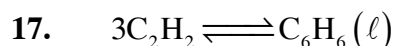
Sol. $[\text{FeCl}_4]^{2-}$ Contain Fe^{+2} in tetrahedral complex. Its configuration is $e_g^{2,1} t_{2g}^{1,1,1}$ it have 4 unpaired electron in $[\text{Co}(\text{ox})_3]^{3-}$ Co^{+3} have configuration $t_{2y}^{2,2,2} e_g^{0,0}$ MnO_4^{2-} have Mn in +6 oxidation state and configuration of Mn is $e_g^{1,0} t_{2g}^{0,0,0}$

15. α -sulphur, β -Sulphur, $\text{S}_2 \rightarrow$ find how many are paramagnetic**Ans. (1)**

Sol. In S_2 , like O_2 two unpaired electron are present, α & β - sulphur have S_8 ring which are diamagnetic

16. Which of the following can be used for clotting of blood efficiently?(1) NaHCO_3 (2) FeCl_3 (3) FeSO_4 (4) $\text{Mg}(\text{HCO}_3)_2$ **Ans. (2)**

Sol. Blood is a negative charged Sol. Therefore according hardy-Schulz rule Fe^{+3} cation have highest coagulation power. Therefore FeCl_3 can be used for clotting of blood efficiently.



given that

$$G_m^\circ (\text{C}_2\text{H}_2) = 2.4 \times 10^5 \text{ J}$$

$$G_m^\circ (\text{C}_6\text{H}_6) = -1.4 \times 10^5 \text{ J}$$

Find $\log_{10} K$ at 25°C **Ans. (150.72)**

Sol. $\Delta G^\circ = (G_M^\circ)_{C_6H_6} - 3(G_M^\circ)_{C_2H_2}$

$$= -1.4 \times 10^5 - 3 \times 2.4 \times 10^5$$

$$= -8.6 \times 10^5 \text{ Joule}$$

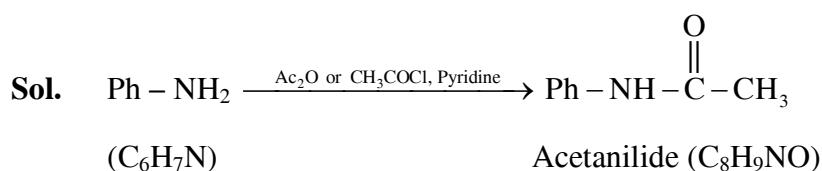
$$-2.303RT \log_{10} k = -8.6 \times 10^5$$

$$-2.303 \times 8.314 \times 298 \log_{10} k = -8.6 \times 10^5$$

$$\log_{10} k = 150.72$$

- 18.** 1.86 gm of aniline is converted into acetanilide with 90% efficiency. Mass of acetanilide formed is $[X] \times 10^{-2}$ gm

Ans. 243×10^{-2}



1.86 g

Molar mass = 93

Molar mass = 135

* 93 g aniline produces 135 g acetanilide

$$1.86 \text{ g aniline produces } \frac{135 \times 1.86}{93} = 2.70 \text{ g}$$

* At 90% efficiency of reaction it produces $= \frac{2.70 \times 90}{100} = 2.43 \text{ g}$

Ans. 243×10^{-2}

- 19.** Freezing point of C_6H_6 (ℓ) is 5.5°C . If 10g of C_4H_{10} is mixed with 200g of C_6H_6 (ℓ). Calculate freezing point of solution $k_f = 5.12^\circ\text{C/m}$.

Ans. (1.09°C)

Sol. $\Delta T_f = k_f \times m$

$$= 5.12 \times \frac{10}{58} \times \frac{1000}{200} = 4.41^\circ\text{C}$$

$$\Delta T_f = (T_f)_{\text{Solvent}} - (T_f)_{\text{Solution}}$$

$$4.41^\circ\text{C} = 5.5 - (T_f)_{\text{Solution}}$$

$$(T_f)_{\text{Solution}} = 5.5 - 4.41 = 1.09^\circ\text{C}$$

20. De-broglie's wavelength of a proton and an α -particle is same. Calculate ratio of their velocities

Ans. (4)

Sol. $\lambda_p = \lambda_\alpha$

$$\frac{h}{m_p v_p} = \frac{h}{m_\alpha v_\alpha}$$

$$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} \quad \because m_\alpha = 4 m_p$$

$$\frac{v_p}{v_\alpha} = \frac{4m_p}{m_p} = 4$$

Ans. 4

21. If $[H^+]$ changed from 1M to 10^{-4} M

Find change in electrode potential $E^\circ_{MnO_4^-/Mn^{+2}}$, $\left(\frac{RT}{F} = 0.059\right)$

[Assume $[MnO_4^-] = [Mn^{+2}] = 1M$]

Ans. 0.3776 V

Sol. $5e^- + 8H^+ + \underset{1M}{MnO_4^-} \longrightarrow \underset{1M}{Mn^{+2}} + 4H_2O$

$$E_1 = E^\circ - \frac{0.59}{5} \log_{10} \left[\frac{1}{[H^+]^8} \times \frac{[Mn^{+2}]}{[MnO_4^-]} \right]$$

$$= E^\circ - \frac{0.059}{5} \log_{10} \left[\frac{1}{(1)^8} \right] = E^\circ$$

$$E_2 = E^\circ - \frac{0.059}{5} \log_{10} \left[\frac{1}{(10^{-4})^8} \times \frac{[Mn^{+2}]}{[MnO_4^-]} \right]$$

$$= E^\circ - \frac{0.059}{5} \log_{10} [10^{32}]$$

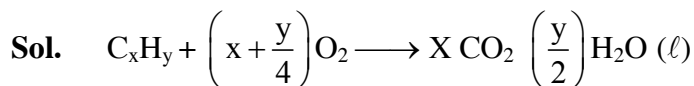
$$= E^\circ - \frac{0.059}{5} \times 32$$

$$E_1 - E_2 = E^\circ - E^\circ + \frac{0.059}{5} \times 32$$

$$= 0.3776 \text{ V}$$

22. V ml of a hydrocarbon C_xH_y requires 6V ml of oxygen for complete combustion & forms 4V ml of CO_2 . Determine y

Ans. 8



Volume-Volume V 6V 4V

Analysis

$$\frac{V_{C_xH_y}}{1} = \frac{V_{CO_2}}{x}$$

$$\frac{v}{1} = \frac{4v}{x} \quad x = 4$$

$$\frac{v_{C_xH_y}}{1} = \frac{V_{O_2}}{x + \frac{y}{4}}$$

$$\frac{V}{1} = \frac{6V}{x + \frac{y}{4}}$$

$$x + \frac{y}{4} = 6$$

$$4 + \frac{y}{4} = 6$$

$$\frac{y}{4} = 2$$

$$y = 8$$

Formula C_4H_8

23. Sucrose $\xrightarrow{\text{I order}}$ Glucose + Fructose

$$t_{1/2} = 3.33 \text{ hour}$$

f = fraction remaining of sucrose at 9 hour.

$$\text{Find out value of } 100 \times \log \left(\frac{1}{f} \right) \quad \left[\log_{10} 2 = 0.3 \right]$$

Ans. (81)

Sol. $f = \frac{1}{2^n}$ $n = \frac{\text{time given}}{t_{1/2}} = \frac{9 \text{ hr}}{3.33 \text{ hr}} = 2.7$

$$= \frac{1}{2^{2.7}}$$

$$\log \frac{1}{f} = \log 2^{2.7} = 2.7 \times 0.3 = 0.81$$

$$100 \times \log_{10} \left(\frac{1}{f} \right) = 100 \times 0.81 = 81$$

Ans. 81

- 24.** Determine volume occupied by 4.75g acetylene gas at 740 mmHg pressure & 50°C temperature
 $R = 0.0826 \text{ Latm/mol k (in L)}$

Ans. (5)

Sol.
$$V = \frac{nRT}{P} = \frac{\left(\frac{4.75}{26} \right) \times 0.0826 \times 323}{\left(\frac{740}{760} \right)} \approx 5\text{L}$$

MATHEMATICS

1. Find the value of ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + \dots + {}^nC_2) = ?$

- (1) $\frac{n(n+1)(2n-1)}{6}$ (2) $\frac{n(n+1)(2n+1)}{6}$
 (3) $\frac{(n-1)n(n+1)}{6}$ (4) $\frac{n(n+1)}{2}$

Ans. (2)

Sol. $S = {}^2C_2 + {}^3C_2 + \dots + {}^nC_2 = {}^{n+1}C_3$

$$\therefore {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3 = {}^{n+2}C_3 + {}^{n+1}C_3$$

$$= \frac{(n+1)!}{3!(n-1)!} + \frac{(n+1)!}{3!(n-2)!}$$

$$= \frac{(n+2)(n+1)n}{6} + \frac{(n+1)(n)(n-1)}{6} = \frac{n(n+1)}{6} (2n+1)$$

2. If A and B are subset s of $X = \{1,2,3,4,5\}$ then find the probability such that $n(A \cap B) = 2$.

- (1) $\frac{65}{2^7}$ (2) $\frac{65}{2^9}$ (3) $\frac{35}{2^9}$ (4) $\frac{135}{2^9}$

Ans. (4)

Sol. Required probability

$$= \frac{{}^5C_2 \times 3^3}{4^5}$$

$$= \frac{10 \times 27}{2^{10}} = \frac{135}{2^9}$$

3. Given $f(0) = 1$, $f(2) = e^2$ also $f'(x) = f'(2-x)$, then the value of $\int_0^2 f(x)dx$ is

- (1) $1 - e^2$ (2) $1 + e^2$ (3) e (4) e^2

Ans. (2)

Sol. $f'(x) = f'(2-x)$

On integrating both side $f(x) = -f(2-x) + c$

put $x = 0$

$$f(0) + f(2) = c \quad \Rightarrow \quad c = 1 + e^2$$

$$\Rightarrow f(x) + f(2-x) = 1 + e^2 \dots (i)$$

$$I = \int_0^2 f(x) dx = \int_0^1 \{f(x) + f(2-x)\} dx = (1 + e^2)$$

4. A curve $y = f(x)$ passing through the point $(1,2)$ satisfies the differential equation $x \frac{dy}{dx} + y = bx^4$

such that $\int_1^2 f(y)dy = \frac{62}{5}$. The value of b is

- (1) 10 (2) 11 (3) $\frac{32}{5}$ (4) $\frac{62}{5}$

Ans. (1)

Sol. $\frac{dy}{dx} + \frac{y}{x} = bx^3$

I.F. = $e^{\int \frac{dx}{x}} = x$

$\therefore yx = \int bx^4 dx = \frac{bx^5}{5} + C$

Passes through $(1,2)$, we get

$2 = \frac{b}{5} + C$..(i)

Also, $\int_1^2 \left(\frac{bx^4}{5} + \frac{C}{x} \right) dx = \frac{62}{5}$

$\Rightarrow \frac{b}{25} \times 32 + C \ln 2 - \frac{b}{25} = \frac{62}{5}$

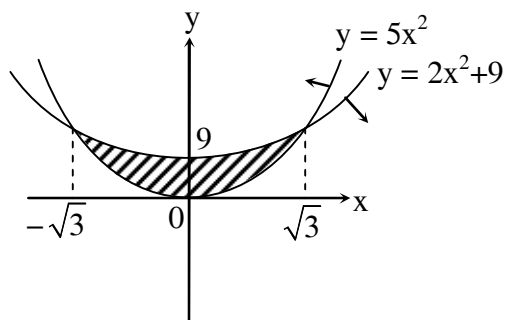
$\Rightarrow C = 0$ & $b = 10$

5. The area of the region defined by $5x^2 \leq y \leq 2x^2 + 9$ is

- (1) $6\sqrt{3}$ (2) $12\sqrt{3}$ (3) $18\sqrt{3}$ (4) $9\sqrt{3}$

Ans. (2)

Sol.



Required area

$= 2 \int_0^{\sqrt{3}} (2x^2 + 9 - 5x^2) dx$

$= 2 \int_0^{\sqrt{3}} (9 - 3x^2) dx$

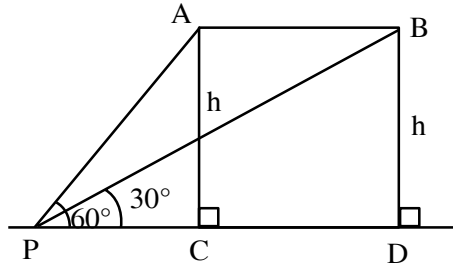
$= 2 \left[9x - x^3 \right]_0^{\sqrt{3}} = 12\sqrt{3}$

6. A aeroplane is flying horizontally with speed of 432 km/hr at height h meter from ground. Its angle of elevation from a point on ground is 60° . After 20 sec its angle of elevation from same point is 30° then the height ' h ' is equal to

- (1) $1200\sqrt{3}$ (2) $600\sqrt{3}$ (3) $1800\sqrt{3}$ (4) $1000\sqrt{3}$

Ans. (1)

Sol.



$$v = 432 \times \frac{1000}{60 \times 60} \text{ m/sec} = 120 \text{ m/sec}$$

$$\text{Distance } AB = v \times 20 = 2400 \text{ meter}$$

In $\triangle PAC$

$$\tan 60^\circ = \frac{h}{PC} \Rightarrow PC = \frac{h}{\sqrt{3}}$$

In $\triangle PBD$

$$\tan 30^\circ = \frac{h}{PD} \Rightarrow PD = \sqrt{3}h$$

$$PD = PC + CD$$

$$\sqrt{3}h = \frac{h}{\sqrt{3}} + 2400 \Rightarrow \frac{2h}{\sqrt{3}} = 2400$$

$$h = 1200\sqrt{3} \text{ meter}$$

7. A curve $y = ax^2 + bx + c$ passing through the point (1, 2) has slope at origin equal to 1. then ordered triplet (a, b, c) may be

- (1) (1, 1, 0) (2) $\left(\frac{1}{2}, 1, 0\right)$ (3) $\left(-\frac{1}{2}, 1, 1\right)$ (4) (2, -1, 0)

Ans. (1)

Sol. $2 = a + b + c$ (i)

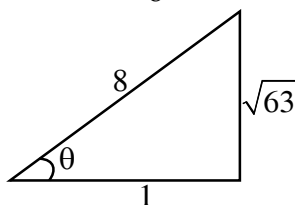
$$\frac{dy}{dx} = 2ax + b \Rightarrow \left. \frac{dy}{dx} \right|_{(0,0)} = 1$$

$$\Rightarrow b = 1 \Rightarrow a + c = 1$$

8. The value of $\tan \left(\frac{1}{4} \sin^{-1} \frac{\sqrt{63}}{8} \right)$ is
- (1) $\frac{1}{\sqrt{7}}$ (2) $\frac{1}{\sqrt{5}}$ (3) $\frac{2}{\sqrt{3}}$ (4) none of these

Ans. (1)

Sol. Let $\sin^{-1} \frac{\sqrt{63}}{8} = \theta \Rightarrow \sin \theta = \frac{\sqrt{63}}{8}$



$$\tan \left(\frac{1}{4} \sin^{-1} \frac{\sqrt{63}}{8} \right) = \tan \left(\frac{\theta}{4} \right) = \frac{1 - \cos \frac{\theta}{2}}{\sin \frac{\theta}{2}} = \frac{1 - \sqrt{\frac{1 + \cos \theta}{2}}}{\sqrt{\frac{1 - \cos \theta}{2}}} = \frac{1 - \frac{3}{4}}{\frac{\sqrt{7}}{4}} = \frac{1}{\sqrt{7}}$$

9. The value of $\int_1^3 [x^2 - 2x - 2] dx$ ([.] denotes greatest integers function)

- (1) -4 (2) -5 (3) $-1 - \sqrt{2} - \sqrt{3}$ (4) $1 - \sqrt{2} - \sqrt{3}$

Ans. (3)

Sol. $I = \int_1^3 -3 dx + \int_1^3 [(x-1)^2] dx$ $x - 1 = t; dx = dt$

$$I = (-6) + \int_0^2 [t^2] dt$$

$$I = -6 + \int_0^1 0 dt + \int_1^{\sqrt{2}} 1 dt + \int_{\sqrt{2}}^{\sqrt{3}} 2 dt + \int_{\sqrt{3}}^2 3 dt$$

$$I = -6 + (\sqrt{2} - 1) + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$$

$$I = -1 - \sqrt{2} - \sqrt{3}$$

10. Which of the following conic has tangent ' $x + \sqrt{3}y - 2\sqrt{3}$ ' at point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2} \right)$?

- (1) $x^2 + 9y^2 = 9$ (2) $y^2 = \frac{x}{6\sqrt{3}}$ (3) $x^2 - 9y^2 = 10$ (4) $x^2 = \frac{y}{6\sqrt{3}}$

Ans. (1)

Sol. tangent to $x^2 + 9y^2 = a$ at point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2} \right)$ is $x \left(\frac{3\sqrt{3}}{2} \right) + 9y \left(\frac{1}{2} \right) = 9$

$\Rightarrow$ option (1) is true

11. The negation of the statement $\sim p \wedge (p \vee q)$ is

- (1) $p \wedge \sim q$ (2) $p \vee \sim q$ (3) $\sim p \wedge q$ (4) $\sim p \vee \sim q$

Ans. (2)

Sol. $\sim(\sim p \wedge (p \vee q))$

$$= \sim((\sim p \wedge p) \vee (\sim p \wedge q))$$

$$= \sim(\sim p \wedge q) = p \vee \sim q$$

12. Equation of plane passing through (1, 0, 2) and line of intersection of planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{i} - 2\hat{j}) = -2$ is

- (1) $\vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$ (2) $\vec{r} \cdot (3\hat{i} + 10\hat{j} + 3\hat{k}) = 7$
 (3) $\vec{r} \cdot (\hat{i} + \hat{j} - 3\hat{k}) = 4$ (4) $\vec{r} \cdot (\hat{i} + 4\hat{j} - \hat{k}) = -7$

Ans. (1)

Sol. Plane passing through intersection of plane is

$$\{\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = -1\} + \lambda \{\vec{r} \cdot (\hat{i} - 2\hat{j}) + 2\} = 0$$

Passes through $\hat{i} + 2\hat{k}$, we get

$$(3 - 1) + \lambda(\lambda + 2) = 0 \quad \Rightarrow \quad \lambda = -\frac{2}{3}$$

$$\text{Hence, equation of plane is } 3\{\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1\} - 2\{\vec{r} \cdot (\hat{i} - 2\hat{j}) + 2\} = 0$$

$$\Rightarrow \vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$$

13. A is 3×3 square matrix and B is 3×3 skew symmetric matrix and X is a 3×1 matrix, then equation $(A^2B^2 - B^2A^2)X = 0$ (Where O is a null matrix) has/have

- (1) Infinite solution (2) No Solution
 (3) Exactly one solution (4) Exactly two solution

Ans. (1)

Sol. $A^T = A$, $B^T = -B$

$$\text{Let } A^2B^2 - B^2A^2 = P$$

$$\begin{aligned} P^T &= (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T \\ &= (B^2)^T (A^2)^T - (A^2)^T (B^2)^T \\ &= B^2A^2 - A^2B^2 \end{aligned}$$

$$\Rightarrow P \text{ is skew-symmetric matrix}$$

$$\Rightarrow |P| = 0$$

Hence $PX = 0$ have infinite solution

14. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, $f(0) = 1$ and $f'(0) = 2$ then $f(1)$ belongs to interval

(1) $[6, 9]$

(2) $[9, 12]$

(3) $[8, 10]$

(4) $[5, 7]$

Ans. (1)

Sol. Given $f(x) f''(x) - (f'(x))^2 = 0$

$$\text{Let } h(x) = \frac{f(x)}{f'(x)}$$

$$\Rightarrow h'(x) = 0$$

$$\Rightarrow h(x) = k$$

$$\Rightarrow \frac{f(x)}{f'(x)} = k$$

$$\Rightarrow f'(x) = k f'(x)$$

$$\Rightarrow f(x) = k f'(0)$$

$$\Rightarrow 1 = k(2) \Rightarrow k = \frac{1}{2}$$

$$\text{New } f(x) = \frac{1}{2} f'(x) \Rightarrow \int 2dx = \int \frac{f'(x)}{f(x)} dx$$

$$\Rightarrow 2x = \ln |f(x)| + C$$

$$\text{As } f(0) = 1 \Rightarrow C = 0$$

$$\Rightarrow 2x = \ln |f(x)| \Rightarrow f(x) = \pm e^{2x}$$

$$\text{As } f(0) = 1 \Rightarrow f(x) = e^{2x} \Rightarrow f(1) = e^2$$

15. Find a point on the curve $y = x^2 + 4$ which is at shortest distance from the line $y = 4x - 1$.

(1) (2,8)

(2) (1,5)

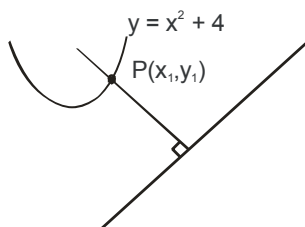
(3) (3,13)

(4) (-1,5)

Ans. (1)

Sol. $\left. \frac{dy}{dx} \right|_p = 4$

$$\therefore 2x_1 = 4$$



$$\Rightarrow x_1 = 2$$

$$\therefore \text{Point will be } (2,8)$$

16. Let $f(x) = \begin{cases} -55x & ; \quad x < -5 \\ 2x^3 - 3x^2 - 120x & ; \quad -5 \leq x < 4 \\ 2x^3 - 3x^2 - 36x + 10 & ; \quad x \geq 4 \end{cases}$

Then interval in which $f(x)$ is monotonically increasing is

- (1) $(-5, -4) \cup (4, \infty)$ (2) $(-\infty, -4) \cup (5, \infty)$
 (3) $(-5, 4) \cup (5, \infty)$ (4) $(-5, -4) \cup (3, \infty)$

Ans. (1)

Sol. $f'(x) = \begin{cases} -55 & ; \quad x < -5 \\ 6(x^2 - x - 20) & ; \quad -5 < x < 4 \\ 6(x^2 - x - 6) & ; \quad x > 4 \end{cases}$

$$f'(x) = \begin{cases} -55 & ; \quad x < -5 \\ 6(x-5)(x+4) & ; \quad -5 < x < 4 \\ 6(x-3)(x+2) & ; \quad x > 4 \end{cases}$$

Hence, $f(x)$ is monotonically increasing is $(-5, -4) \cup (4, \infty)$

17. If a, b, c are in A.P. & centroid of the triangle with vertices $(a, c), (a, b), (2, b)$ is $\left(\frac{10}{3}, \frac{7}{3}\right)$ and

α, β are roots of the equation $ax^2 + bx + 1 = 0$, then $\alpha^2 + \beta^2 - \alpha\beta$ equals

- (1) $-\frac{71}{256}$ (2) $\frac{71}{256}$ (3) $\frac{69}{256}$ (4) $-\frac{69}{256}$

Ans. (1)

Sol. $2b = a + c$

$$\frac{2a+2}{3} = \frac{10}{3} \text{ and } \frac{2b+c}{3} = \frac{7}{3}$$

$$\Rightarrow a = 4 \quad \left. \begin{matrix} 2b+c=7 \\ 2b-c=4 \end{matrix} \right\} \text{ solving,}$$

$$b = \frac{11}{4} \quad c = \frac{3}{2}$$

$$\therefore \text{ Quadratic Equation is } 4x^2 + \frac{11}{4}x + 1 = 0$$

$$\therefore \text{ The value of } (\alpha + \beta)^2 - 3\alpha\beta = \frac{121}{256} - \frac{3}{4} = -\frac{71}{256}$$

18. Given $a + \alpha = 1$, $b + \beta = 2$ and $\alpha f(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ then value of $\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}}$

Ans. 2

Sol. $\alpha f(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ (i)

$$x \rightarrow \frac{1}{x}$$

$$\alpha f\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x$$
 (ii)

(i) + (ii)

$$(a + \alpha) \left[f(x) + f\left(\frac{1}{x}\right) \right] = \left(x + \frac{1}{x} \right) (b + \beta)$$

$$\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{2}{1} = 2$$

19. Find the maximum value of 'k' for which the maximum value of variance of 10 elements is 10 in which 9 values are 1 and one value of is k. (Where k is integer)

Ans. 11

Sol. $\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n} \right)^2$

$$\sigma^2 = \frac{(9 + k^2)}{10} - \left(\frac{9 + k}{10} \right)^2 < 10$$

$$(90 + k^2)10 - (81 + k^2 + 8k) < 1000$$

$$90 + 10k^2 - k^2 - 18k - 81 < 1000$$

$$9k^2 - 18k + 9 < 1000$$

$$(k - 1)^2 < \frac{1000}{9} \Rightarrow k - 1 < \frac{10\sqrt{10}}{3}$$

$$k < \frac{10\sqrt{10}}{3} + 1$$

Maximum integral value of k = 11

- 20.** Distance of P(x, y) from (5,0) is thrice as distance of P(x, y) from (-5,0). If locus of P is circle with radius 'r' then find the value of $4r^2$.

Ans. 56.25

Sol. Internal point which divide (5,0) & (-5,0) in the ratio 3 : 1 is $\left(\frac{-5}{2}, 0\right)$ External point which divide (5,0) & (-5,0) in the ratio 3 : 1 is (-10,0)

$$2r = \left(\frac{-5}{2} + 10\right) = \frac{15}{2} = 7.5$$

$$(2r)^2 = 56.25$$

- 21.** Four numbers whose sum is $\frac{65}{12}$ are in G.P. Sum of their reciprocals is $\frac{65}{18}$ and product of first three of them is 1. If third term is α then find value of 2α .

Ans. 3

Sol. a, ar, ar^2 , ar^3

$$a + ar + ar^2 + ar^3 = \frac{65}{12} \quad \dots\dots (i)$$

$$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$$

$$\frac{1}{a} \left(\frac{r^3 + r^2 + r + 1}{r^3} \right) = \frac{65}{18} \quad \dots\dots (ii)$$

$$\frac{(i)}{(ii)}, a^2 r^3 = \frac{18}{12} = \frac{3}{2}$$

$$a^3 r^3 = 1 \Rightarrow a \left(\frac{3}{2} \right) = 1 \Rightarrow a = \frac{2}{3}$$

$$\frac{4}{9} r^3 = \frac{3}{2} \Rightarrow r^3 = \frac{3^3}{2^3} \Rightarrow r = \frac{3}{2}$$

$$\alpha = ar^2 = \frac{2}{3} \cdot \left(\frac{3}{2} \right)^2 = \frac{3}{2}$$

$$2\alpha = 3$$

- 22.** There are 10 students $S_1, S_2, \dots\dots S_{10}$. Find the number of ways to form 3 groups G_1, G_2, G_3 such that all groups has at least 1 member and group G_3 has almost 3 members

Ans. 26650

A	B	C
1	8	1
2	7	1
$\vdots$	$\vdots$	$\vdots$
6	1	3

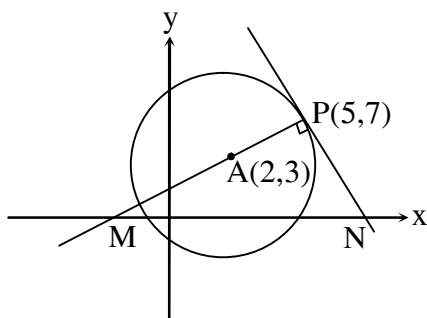
Sol.

$$\begin{aligned} \text{Ways to distribute in groups} &= {}^{10}C_1({}^9C_1 + \dots\dots {}^9C_8) + {}^{10}C_2({}^8C_1 + \dots\dots + {}^8C_7) + {}^{10}C_3({}^7C_1 + \dots\dots + {}^7C_6) \\ &= 10(510) + 45(254) + 120(126) \\ &= 26650 \end{aligned}$$

23. At point P(5, 7) on circle $(x - 2)^2 + (y - 3)^2 = 25$ a tangent and a normal is drawn. The area of triangle formed by this tangent normal with x axis is λ then 24λ is

Ans. 1225

Sol.



equation of normal at P

$$(y - 7) = \left(\frac{7-3}{5-2} \right) (x - 5)$$

$$3y - 21 = 4x - 20$$

$$\Rightarrow 4x - 3y + 1 = 0 \quad \dots\dots\dots (i)$$

$$\Rightarrow M\left(-\frac{1}{4}, 0\right)$$

equation of tangent at P

$$(y - 7) = -\frac{3}{4} (x - 5)$$

$$4y - 28 = -3x + 15$$

$$\Rightarrow 3x + 4y = 43 \quad \dots\dots\dots (ii)$$

$$\Rightarrow N\left(\frac{43}{3}, 0\right)$$

$$\text{hence ar}(\triangle PMN) = \frac{1}{2} \times MN \times 7$$

$$l = \frac{1}{2} \times \frac{175}{12} \times 7$$

$$\Rightarrow 24\lambda = 1225$$