PATHFINDER®
Where Aspiration Meets Success

JEE(Main) : 2021

(Questions with Answer)

Date : 24th February 2021 (Shift-1) | Time : 9.00 a.m. to 12.00 p.m|

Duration : 3 Hours | Max. Marks : 300

(Memory Based Question)

PHYSICS

CHEMISTRY

MATHEMATICS

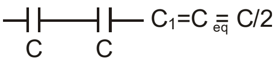
PHYSICS

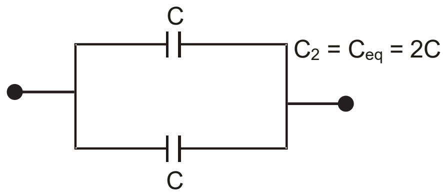
1. If two identical capacitor once connected in series ($C_{eq} = C_1$) and once connected in parallel ($C_{eq} = C_2$) then find ratio $\frac{C_1}{C_2}$.

(1) $\frac{3}{4}$ (2) $\frac{5}{6}$ (3) $\frac{4}{3}$ (4) $\frac{1}{4}$

Ans. (4)

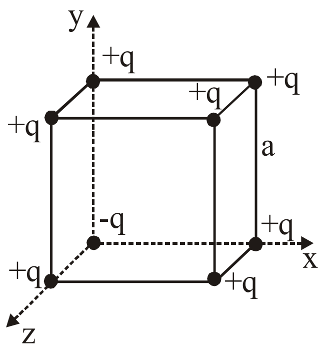
Sol.





$$\boxed{\frac{C_1}{C_2} = \frac{1}{4}}$$

2.



Find electric field at centre of cube ?

(1) $\frac{-2q}{\pi 3\sqrt{3}\varepsilon_0 a^2}\left[\hat{i}+\hat{j}+\hat{k}\right]$ (2) $\frac{q}{\pi 3\sqrt{3}\varepsilon_0 a^2}\left[\hat{i}+\hat{j}+\hat{k}\right]$

(3) $\frac{-q}{\pi 3\sqrt{3}\varepsilon_0 a^2}\left[\hat{i}+\hat{j}+\hat{k}\right]$ (4) $\frac{2q}{\pi 3\sqrt{3}\varepsilon_0 a^2}\left[\hat{i}+\hat{j}+\hat{k}\right]$

Ans. **(1)**

Sol. If we consider two point charges +q and –q at position of –q charge, then after interchanging –q charge with +q charge, net electric field at centre of cube is zero due to symmetry. Now remaining charges are –2q so net electric field at centre is $\left(\frac{-8kq}{3a^2}\right)$.

3. In a transistor if increase in emitter current is 4 mA and corresponding increase in collector current is 3.5 mA. Find β.

(1) .875 (2) .5 (3) 7 (4) 1

Ans. **(3)**

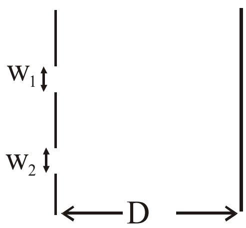
Sol. $\Delta I_E = 4$

$\Delta I_C = 3.5$

$$\alpha = \frac{\Delta I_C}{\Delta I_E} = \left(\frac{3.5}{4}\right) = \left(\frac{7}{8}\right)$$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{\frac{7}{8}}{1-\frac{7}{8}} = 7$$

4. If $\frac{w_1}{w_2} = \frac{1}{3}$ (w is width) and its given that amplitude is proportional to w. Find $\frac{I_{max}}{I_{min}}$.



(1) 1 : 4 (2) 3 : 1 (3) 4 : 1 (4) 2 : 1

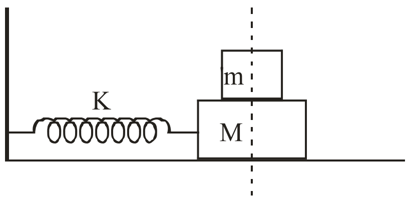
Ans. **(3)**

Sol. $\frac{A_1}{A_2} = \frac{1}{3}$ $A_1 = x$, $A_2 = 3x$

$$\frac{I_{max}}{I_{min}} = \left(\frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}\right) = \frac{(4x)^2}{(2x)^2} = \frac{16}{4} = 4:1$$

5. If a small block of mass m is placed gently on M when it is passing through its mean position.

Find it's new amplitude of SHM, if initial amplitude is A.



(1) $\sqrt{\frac{M}{M+m}}$ (A) (2) $\sqrt{\frac{M+m}{M}}$ (A) (3) $\sqrt{\frac{M+m}{2M}}$ (A) (4) $\sqrt{\frac{M}{M-m}}$ (A)

Ans. **(1)**

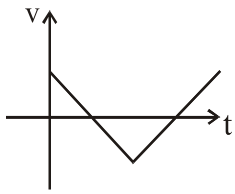
Sol. Velocity at mean position is = $A\omega$

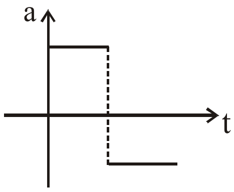
Conserving momentum $MA\omega_O = (M + m)V'$

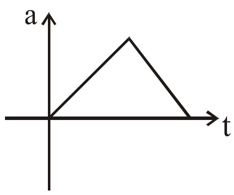
$$V' = \frac{MA\omega_O}{M+m} = (A')\sqrt{\frac{K}{M+m}}$$

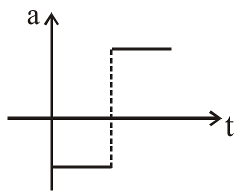
$$A' = \frac{MA\sqrt{\frac{K}{M}}}{M+m} \times \sqrt{\frac{M+m}{K}} = \sqrt{\frac{M}{(M+m)}}A$$

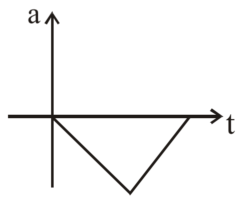
6. From given velocity time graph which of the following is acceleration time graph.



(1) 

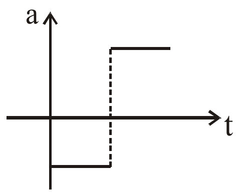
(2) 

(3) 

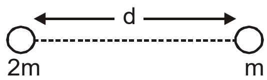
(4) 

Ans. **(3)**

Sol.
$$\begin{array}{l|l|} v=-mt+C & v=mt-C \\ \frac{dv}{dt}=-m & \frac{dv}{dt}=m \end{array}$$



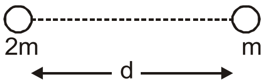
7. Two stars are rotating about their common center of mass. Find the period of revolution of mass m about center of mass.



(1) $2\pi\sqrt{\frac{d^3}{3Gm}}$ (2) $2\pi\sqrt{\frac{3Gm}{d^3}}$ (3) $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$

Ans. **(1)**

Sol.



$$\frac{G(m)(2m)}{d^2} = m\omega^2 \times \frac{2d}{3}$$

$$\frac{2Gm}{d^2} = \omega^2 \times \frac{2d}{3}$$

$$\omega^2 = \frac{3Gm}{d^3}$$

$$\omega = \sqrt{\frac{3Gm}{d^3}} \quad ; \qquad T = 2\pi\sqrt{\frac{d^3}{3Gm}}$$

8. Find relation between modulus

If

$Y \rightarrow$ young's modulus

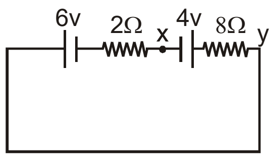
$K \rightarrow$ Bulk modulus

$\eta \rightarrow$ modulus of rigidity

(1) $K = \frac{\eta Y}{9\eta - 3Y}$ (2) $Y = \frac{\eta K}{9\eta - 3K}$ (3) $K = \frac{\eta Y}{9\eta + 3Y}$ (4) $Y = \frac{\eta K}{9\eta + 3K}$

Ans. (1)

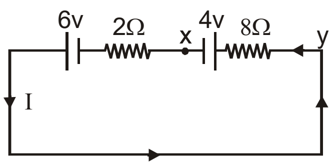
9. Circuit is given find $|V_x - V_y|$



(1) 5.6 volt (2) 3.6 volt (3) 10 v (4) 240 volt

Ans. (1)

Sol. Current $I = \frac{6-4}{10} = \frac{1}{5}A$



$v_x + 4 + 8 \times \frac{1}{5} = v_y$

$\boxed{v_x - v_y = -5.6v}$

10. If work $= \alpha \cdot \beta^2 \cdot e^{-\frac{x^2}{\alpha \cdot KT}}$,

where, K = Boltzmann constant, T = temp, then find dimension of β

(1) $M^1L^1T^{-2}$ (2) $M^{-1}L^{-1}T^2$ (3) $M^1L^2T^{-2}$ (4) $M^2L^1T^2$

Ans. (1)

Sol. $\frac{x^2}{\alpha KT} = \text{dimensionless}$

$\frac{L^2}{KT} \Rightarrow \alpha$

$\alpha \Rightarrow \frac{L^2}{v^2 M} = L^2M^{-1}L^{-2}T^2 = M^{-1}T^2$

work $= \alpha \cdot \beta^2 \cdot$ (dimensionless)

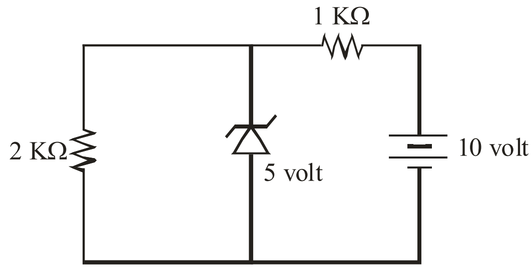
$M^1L^1T^{-2} \cdot L^1 = M^{-1}T^2\beta^2$

$v = \sqrt{\frac{3KT}{M}}$

$\frac{v^2 \cdot M}{3} = KT$

$\beta = M^1L^1T^{-2}$

11. Zener diode is connected in circuit as shown. Find current in $2k\Omega$ resistance.



(1) 2.5mA (2) 30mA (3) 10mA (4) 20mA

Ans. **(1)**

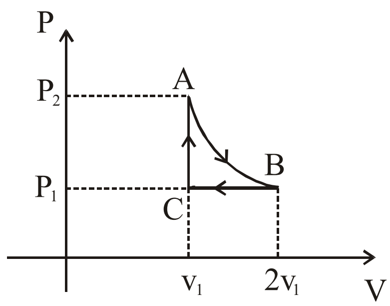
Sol. Zener diode breakdown

$\Rightarrow \quad i = \frac{5}{2\times10^{-3}} = 2.5 \times 10^{-3}$

$x \times 10^{-4} = 2.5 \times 10^{-3}$

$x = 2.5mA$

12. **G**iven AB is Isothermal and BC is Isobaric and CA is isochoric process. Find total work done in cyclic process. (Given $p_1v_1 = nRT$).



(1) $nRT[\ell n2 - 2]$ (2) $2nRT[\ell n2 - 1/2]$ (3) $nRT[\ell n2 - 1/2]$ (4) $nRT[\ell n2 + 2]$

Ans. **(2)**

Sol. $W_{AB} = 2\ P_1V_1\ \ell n\ 2$

$W_{BC} = -P_1V_1$

$W_{CA} = 0$

$W_{ABCA} = (2\ P_1V_1\ \ell n\ 2 - P_1V_1)$

$= nRT\ (2\ \ell n\ 2 - 1)$

13. A side of cube a is made from 6 identical sheet. If coefficient of linear expansion is α and temperature is now made $T + \Delta T$ from temperature T then find change in volume

(1) $\Delta V = 3a^3\ \alpha\ \Delta T$ (2) $\Delta V = \frac{4}{3}\pi a^3\ \alpha\ \Delta T$ (3) $\Delta V = 4a^3\ \alpha\Delta T$ (4) $\Delta V = 6a^3\ \alpha\ \Delta T$

Ans. **(1)**

Sol. $\frac{\Delta V}{V} = \gamma\Delta T$

$= 3\alpha\Delta T$

$\Delta V = 3a^3\ \alpha\ \Delta T$

14. If intensity after passing through a polaroid is 100 Lumen. Now this same polaroid is rotated by 30° about it's axis then find new intensity after passing through it.

(1) 50 Lumen (2) 100 Lumen (3) 25 Lumen (4) 65 Lumen

Ans. **(2)**

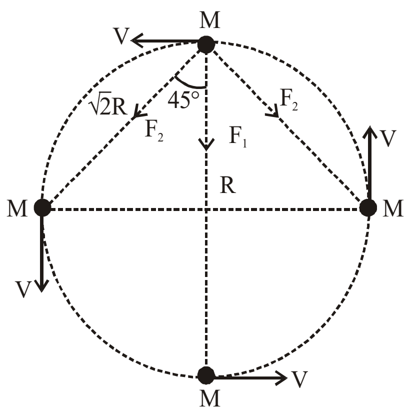
Sol. $I_{P_1} = I_{P_2}$ = 100 Lumen

15. Four particles, each of mass 1 kg and equidistant from each other, move along a circle of radius 1 m under the action of their mutual gravitational attraction. The speed of each particle is :

(1) $\sqrt{G(1+2\sqrt{2})}$ (2) $\frac{1}{2}\sqrt{G(1+2\sqrt{2})}$ (3) $\sqrt{G}$ (4) $\sqrt{2\sqrt{2}G}$

Ans. **(2)**

Sol.



Net force on one particle

$F_{net} = F_1 + 2F_2 \cos 45° =$ Centripetal force

$$\Rightarrow \frac{GM^2}{(2R)^2} + \left[\frac{2GM^2}{(\sqrt{2}R)^2}\cos 45^\circ\right] = \frac{MV^2}{R}$$

$$V = \frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$$

$$v = \frac{1}{2}\sqrt{G(1+2\sqrt{2})}$$

16. Choose the correct options.

Column-I	Column-II
(P) Adiabatic	(i) Pressure constant
(Q) Isothermal	(ii) Volume constant
(R) Isochoric	(iii) Temperature constant
(S) Isobaric	(iv) Heat content constant

from above match the column find out correct options.

(1) P-iv ; Q-iii ; R-ii ; S-i
(2) P-iv ; Q-iii ; R-i ; S-ii
(3) P-iii ; Q-ii ; R-iv ; S-i
(4) P-iv ; Q-ii ; R-iii ; S-i

Ans. **(1)**

Sol. P-iv ; Q-iii ; R-ii ; S-i

17. Relation between radius of curvature & focal length of a convex mirror is.

(1) $f = \frac{R}{2}$ (2) $R = \frac{f}{2}$ (3) 2f = 3R (4) none

Ans. **(1)**

Sol. $\frac{R}{2} = f$

18. I_1 = Moment of Inertia of ring of mass M and radius R about its diameter.

I_2 = Moment of Inertia of Disc of mass M and radius R through central axis perpendicular to plane of ring.

I_3 = Moment of Inertia of solid cylinder mass M and radius R about its axis.

I_4 = Moment of Inertia of solid sphere through its diameter.

(1) $I_1 + I_2 = I_3 + \frac{5}{4}I_4$

(2) $I_1 = I_2 = I_3 < I_4$

(3) $I_1 > I_2 = I_3 = I_4$

(4) $I_1 + I_2 = I_3 + \frac{2}{5}I_4$

Ans. (1)

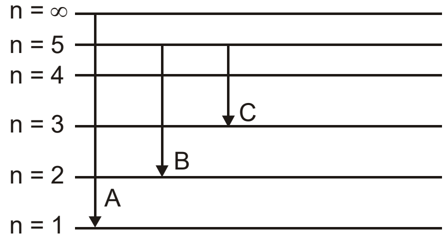
Sol. $I_1 = \frac{MR^2}{2}$

$I_2 = \frac{MR^2}{2}$

$I_3 = \frac{MR^2}{2}$

$I_4 = \frac{2}{5}MR^2$

19. Spectral lines of hydrogen atoms are given is below



Which of the following is correct for spectral lines A, B & C

(1) Series limit of lyman, 3rd line of Balmer, 2nd line of paschan

(2) Series limit of lyman, 2nd line of Balmer, 2nd line of paschan

(3) 2nd line of lyman, 3rd line of Balmer, 2nd line of paschan

(4) Series limit of lyman, 3rd line of Balmer, 3rd line of paschan

Ans. (1)

Sol. A $\rightarrow$ Series limit of laymen

B $\rightarrow$ 3^{rd} line of Balmer

C $\rightarrow$ 2^{rd} line of paschan

20. **Statement-1:** de-Broglie wavelength of proton varies inversely with linear momentum.

Statement-2: As λ increases, energy and momentum increases.

(1) Statement-1 is true and Statement-2 is false.

(2) Statement-1 is false and Statement-2 is true.

(3) Statement-1 is false and Statement-2 is false.

(4) Statement-1 is true and Statement-2 is true.

Ans. **(1)**

Sol. $\lambda = \frac{h}{p}$

21. A 220 volts AC supply is given to the primary circuit of transformer and as output of 12 volts DC is taken out using rectifier. If secondary number of turns was 24, then find the no. of turns in primary coil.

Ans. **440**

Sol. $\frac{N_P}{N_S} = \frac{V_P}{V_S}$

$\frac{N_P}{24} = \frac{220}{12}$; $N_P = 440$

22. A vertical cross-section of plane is $y = \frac{x^2}{4}$, coefficient of friction is 0.5. Find maximum height at which particle can stay (in cm).

Ans. **25 cm**

Sol. $\mu \geq \tan\theta = \frac{dy}{dx} = \frac{dx}{4} = \frac{x}{2}$

$0.5 \geq \frac{x}{2}$

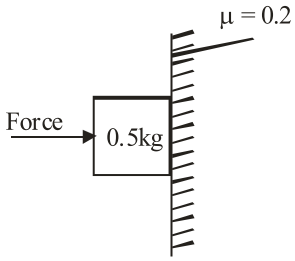
$x \leq 1$

$\sqrt{4y} \leq 1$

$2\sqrt{y} \leq 1$

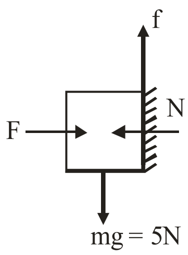
$y \leq \frac{1}{4}$

23. Find out minimum force required to stop block from falling (given g = 10 m/s^2)



Ans. **25**

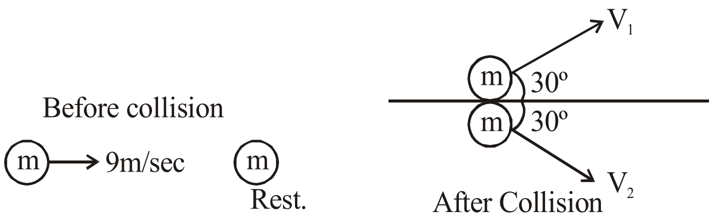
Sol. F = N, f = 0.2 × N



$0.2\ N \le 5$

$N \le 25$

24. Two identical particles of same mass are shown just before collision and just after collision than $\frac{v_1}{v_2}$ is



Ans. **1**

Sol. Using linear momentum conservation in y-direction

$P_i = 0$

$P_f = m \times \frac{1}{2} v_1 - m \times \frac{1}{2} v_2$

$v_1 = v_2$

25. $i = 20t + 8t^2$

Find charge flown during $0 \le t \le 15$

Ans. **11250**

Sol. $\frac{dq}{dt}=(20t+8t^2)$

$\int dq=\int_0^{15}(20t+8t^2)dt$

$\Delta q=\left[20\frac{t^2}{2}+\frac{8t^3}{3}\right]_0^{15}$

$=\frac{20\times(15)^2}{2}+\frac{8\times(15)^3}{3}$

Δq = 11250 C

26. A 100 kg block can be lift by placing a mass m on a piston (hydraulic lift). If diameter of piston placed at one and is increased by 4 times and that of second piston is decreased by 4 times. Now same mas m is placed at piston than how many kg weight can be lift up for the new set up?

Ans. 25600

Sol. Initially $\frac{100g}{A_1}=\frac{mg}{A_2}$ ....(i)

Initially $\frac{Mg}{16A_1}=\frac{mg}{\left(\frac{A_2}{16}\right)}$ ....(ii)

$\frac{100\times16}{M}=\frac{1}{16}$ = M = 25600kg

27. Two planet are resolving about planet with $\frac{T_1}{T_2}=\frac{1}{8}$. Find the ratio of the angular speed $\frac{\omega_1}{\omega_2}$

Ans. **8**

Sol. Ratio of time period

$\frac{T_1}{T_1}=\frac{1}{8}$

$\frac{\frac{2\pi}{\omega_1}}{\frac{2\pi}{\omega_2}}=\frac{1}{8}$

$\frac{\omega_1}{\omega_2}=8$

28. An electromagnetic wave is propagating in medium, where $\mu_r=\varepsilon_r=2$. If speed of light in medium is x × 10^7 m/s. Find out x.

Ans. **15**

Sol. n = $\sqrt{\mu_r\varepsilon_r}=2$

$v=\frac{C}{n}=\frac{3\times10^8}{2}$ = 15 × 10^7 m/s

x = 15

29. $V_m = 20 \sin [100\ \pi t + \frac{\pi}{4}]$

$V_c = 80 \sin [10^4\ \pi t + \frac{\pi}{6}]$

For amplitude modulation wave findout percentage modulation index.

Ans. 25

Sol. $m\% = \frac{A_m}{A_c} \times 100 = \frac{20}{80} \times 100 = 25\%$

30. In AC series R-L-C resonance circuit Resistance R = 6.28 ohm, frequency 10 MH_Z and self inductance $L = 2 \times 10^{-4}$ Henry is given. Find Quality factor of circuit.

Ans. 2000

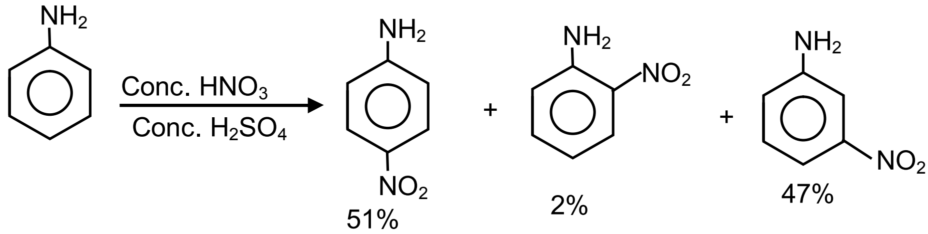
Sol. $Q = \frac{x_L}{R} = \frac{\omega L}{R} = \frac{2\pi f L}{R}$

$Q = \frac{2\pi \times 10^6 \times 10 \times 2 \times 10^{-4}}{6.28} = 2000$

$Q = 2000$

CHEMISTRY

1. What is the reason for the formation of meta product in the following reaction?



(1) Aniline is ortho/para directing

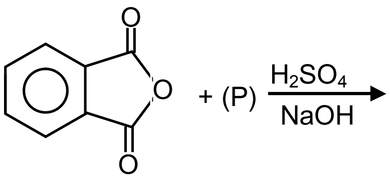
(2) Aniline is meta directing

(3) In acidic medium, aniline is converted into anilinium ion which is ortho/para directing

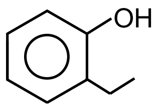
(4) In acidic medium, aniline is converted into anilinium ion which is meta directing

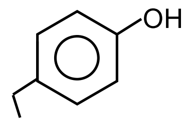
Ans. **(4)**

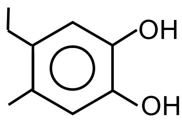
Sol. In acidic medium, aniline is converted into anilinium ion which is meta directing

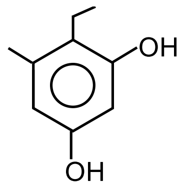
2.  Pink colour

Missing reagent (P) is

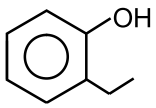
(1) 

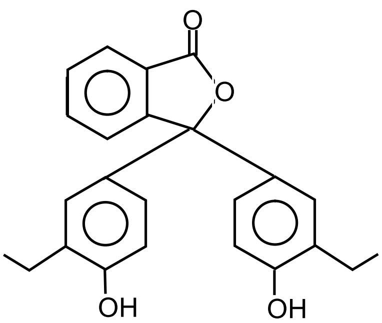
(2) 

(3) 

(4) 

Ans. **(1)**

Sol. P is 



3. Which force is responsible for the stacking of α-helix structure of protein?

(1) H-bonding (2) Ionic bonding (3) Covalent bond (4) Vanderwal forces

Ans. (1)

Sol. Hydrogen bond is responsible for the stacking of α-helix structure of protein.

4. The gas evolved due to anaerobic degradation of vegetation causes?

(1) Global warming and caner

(2) Acid rain

(3) Ozone hole

(4) Metal corrosion

Ans. (1)

Sol. The gas CH_4 evolved due to anaerobic degradation of vegetation which causes global warming and caner.

5. Match the column

(i) Caprolactum	(a) Neoprene
(ii) Acrylo nitrile	(b) Buna N
(iii) 2-chlorobuta-1,3-diene	(c) Nyolon-6
(iv) 2-Methylbuta-1,3-diene	(d) Natural rubber

(1) (i) $\rightarrow$ (b), (ii) $\rightarrow$ (c), (iii) $\rightarrow$ (a), (iv) $\rightarrow$ (d)

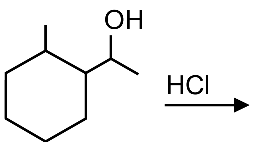
(2) (i) $\rightarrow$ (a), (ii) $\rightarrow$ (c), (iii) $\rightarrow$ (b), (iv) $\rightarrow$ (d)

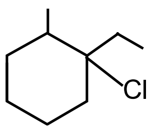
(3*) (i) $\rightarrow$ (c), (ii) $\rightarrow$ (b), (iii) $\rightarrow$ (a), (iv) $\rightarrow$ (d)

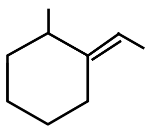
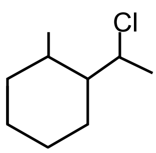
(4) (i) $\rightarrow$ (c), (ii) $\rightarrow$ (a), (iii) $\rightarrow$ (b), (iv) $\rightarrow$ (d)

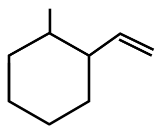
Ans. (3)

6. What is the major product of the following reaction?



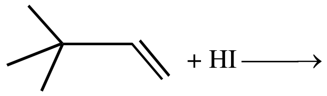
(1) 

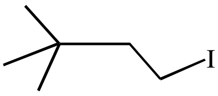
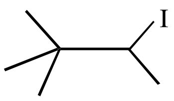
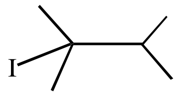
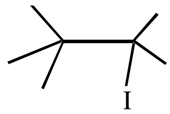
(2) 
(3) 

(4) 

Ans. (1)

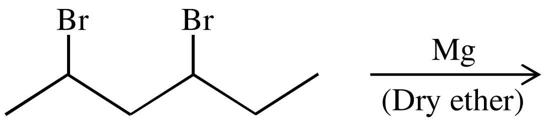
7. What is the major product of following reaction?

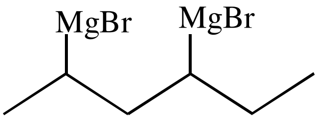


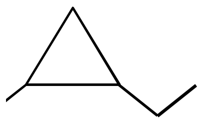
(1)  (2)  (3)  (4) 

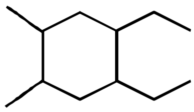
Ans. **(3)**

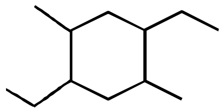
8. Identify the major product?



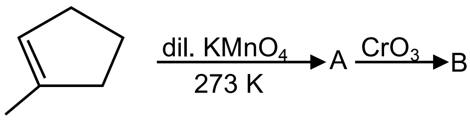
(1) 

(2) 

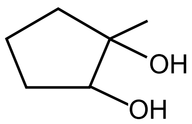
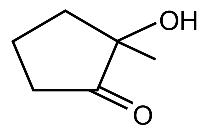
(3) 

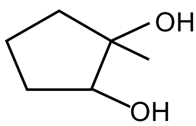
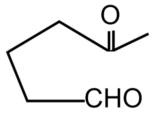
(4) 

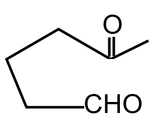
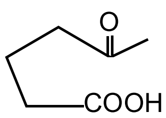
Ans. **(2)**

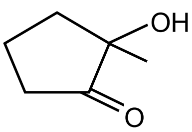
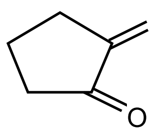
9. 

Product A and B are ?

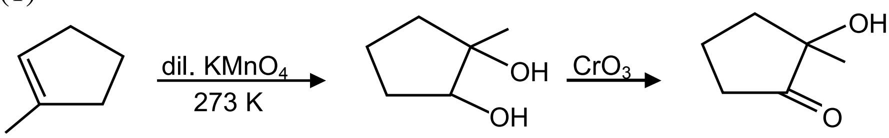
(1) A =  B = 

(2) A =  B = 

(3) A =  B = 

(4) A =  B = 

Ans. (1)

Sol. 

10. $CH_3-CH_2-CH_3 \xrightarrow{(A)} CH_3-CH_2-\overset{\overset{O}{\|}}{C}-H$

Which reagent (A) is used for following given conversion?

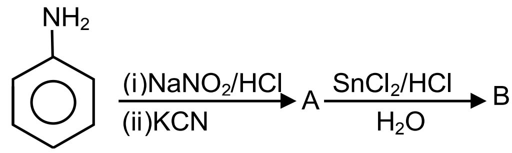
(1) Cu/Δ/ high pressure

(2) Molybdenum oxide

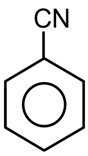
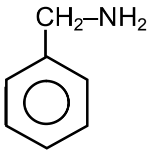
(3) Manganese acetate

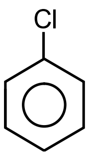
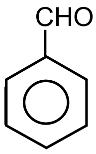
(4) Potassium permanganate

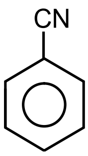
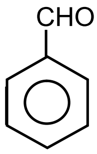
Ans. **(2)**

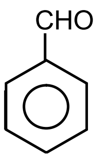
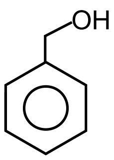
11. 

Find A and B

(1) A =  B = 

(2) A =  B = 

(3) A =  B = 

(4) A =  B = 

Ans. **(3)**

12. Which of the following have both the compound isostructural.

(A) $TiCl_4$, $SiCl_4$ (B) SO_4^{2-}, CrO_4^{2-} (C) NH_3, NO_3^- (D) ClF_3, BCl_3

(1) A,B (2) A,C (3) B,C (4) A,D

Ans. **(1)**

13. Which of the following ores are concentrated by cyanide of group Ist element.

(1) Sphalerite (2) Malachite (3) Calamine (4) Siderite

Ans. **(1)**

14. S-1: Colourless cupric metaborate is converted into cuprous metaborate in luminous flame.

S-2: Cuprous metaborate is formed by reacting copper sulphate with boric anhydride heated in non luminous flame.

(1) S_1 is true and S_2 is false (2) S_1 is flase and S_2 is true

(3) Both are false (4) Both are true.

Ans. (3)

15. (1) $I_2 + H_2O_2 + 2OH^- \longrightarrow 2I^- + 2H_2O + O_2$

(2) $H_2O_2 + HOCl \longrightarrow Cl^- + H_3O^+ + O_2$

(1) H_2O_2 is acting as oxidising agent in both the reaction

(2) H_2O_2 is acting as reducing agent in both the reaction

(3) H_2O_2 is acting as oxidising agent in reaction (1) and as reducing agent in reaction(2)

(4) H_2O_2 is acting as reducing agent in reaction (1) and as oxidising agent in reaction (2)

Ans. (2)

16. $E^o_{M^{2+}/M}$ has positive value for which of the element of 3d transition series.

Ans. (Cu)

17. $Al + NaOH \longrightarrow X \xrightarrow{Y_{(g)}} Z$

Identify X,Y,Z in the above reaction sequence

(1) $X = Na[Al(OH)_4$ $Y = CO_2$ $Z = Al_2O_3.\ x\ H_2O$

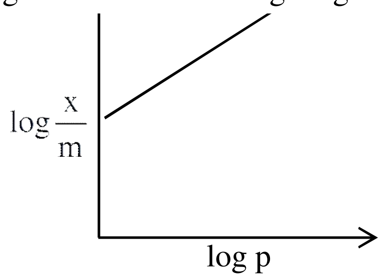
(2) $X = Na[Al(OH)_4$ $Y = SO_2$ $Z = Al_2O_3.\ x\ H_2O$

(3) $X = Al(OH)_3$ $Y = CO_2$ $Z = Al_2O_3$

(4) $Al(OH)_3$ $Y = SO_2$ $Z = Al_2O_3$

Ans. (1)

18. The slope of the straight line given in the following diagram for adsorption is



(1) $\frac{1}{n}$(0 to 1) (2) $\frac{1}{n}$(0.1 to 0.5) (3) log n (4) log $\frac{1}{n}$

Ans. (1)

Sol. Slope $= \frac{1}{n}$ (0 to 1)

19. Composition of gun metal is

(1) Cu, Zn, Sn (2) Al, Mg, Mn, Cu

(3) Cu, Ni, Fe (4) Cu, Sn, Fe

Ans. (1)

20. Arrange the following in the correct order of ionisation potential
Mg, Al, Si, P, S

Ans. Al < Mg < Si < S < P

Sol. Theory

21. $Cl_{2(g)} \rightleftharpoons 2Cl_{(g)}$

For the given reaction at equilibrium moles of $Cl_{2(g)}$ is equal to the moles of $Cl_{(g)}$ and equilibrium pressure is 1atm. if K_p of this reaction is $x \times 10^{-1}$. Find x

Ans. **(5)**

Sol. $Cl_2 \rightleftharpoons 2Cl$

Moles at eq^n: x x

P.P. $\frac{1}{2}$ $\frac{1}{2}$

$$K_P = \frac{P_{Cl}^2}{P_{Cl_2}}$$

$$= \frac{\left(\frac{1}{2}\right)^2}{\frac{1}{2}} = \frac{1}{2} = 0.5$$

$$= 5 \times 10^{-1}$$

$$x = 5$$

22. $S_8 + b\,OH^- \longrightarrow c\,S^{2-} + d\,S_2O_3^{2-} + H_2O$

Find the value of c.

Ans. **(4)**

Sol. $S_8 + 12\,OH^- \longrightarrow 4S^{2-} + 2S_2O_3^{2-} + 6H_2O$

23. Calculate time taken in seconds for 40% completion of first order reaction if rate constant is 3.3×10^{-4} sec^{-1}.

Ans. 1535.3

Sol. $t = \frac{2.303}{K} \log \frac{100}{100 - x}$

$$= \frac{2.303}{3.3 \times 10^{-4}} \log \frac{100}{100 - 40}$$

$$= \frac{2.303}{3.3 \times 10^{-4}} \times 0.22$$

= 1535.3 sec.

24. For a chemical reaction K_{eq} is 100 at 300K, the value of $\Delta_r G$ is $-xR$ Joule at 1 atm pressure. Find the value of x. (Use ln 10 = 2.3)

Ans. 1380

Sol. $\Delta_r G^\circ = -RT\ln K_{eq}$

$= -R \times 300 \times 2 \times 2.3$

$= -1380\ R$

25. $Cu^{2+} + NH_3 \rightleftharpoons [Cu(NH_3)]^{2+}$ $K_1 = 10^4$

$[Cu(NH_3)]^{2+} + NH_3 \rightleftharpoons [Cu(NH_3)_2]^{2+}$ $K_2 = 1.58 \times 10^3$

$[Cu(NH_3)_2]^{2+} + NH_3 \rightleftharpoons [Cu(NH_3)_3]^{2+}$ $K_3 = 5 \times 10^2$

$[Cu(NH_3)_3]^{2+} + NH_3 \rightleftharpoons [Cu(NH_3)_4]^{2+}$ $K_4 = 10^2$

Dissociation constant of $[Cu(NH_3)_4]^{2+}$ is $x \times 10^{-12}$.

Determine x

Ans. 1.26 (Nearest integer = 1)

Sol. $[Cu(NH_3)_4]^{2+} \rightleftharpoons Cu^{2+} + 4NH_3$

$$K = \frac{1}{K_1K_2K_3K_4} = \frac{1}{10^4 \times 1.58 \times 10^3 \times 5 \times 10^2 \times 10^2}$$

$= 1.26 \times 10^{-12} = 1.26$

26. $CH_2ClCOOH$ is dissolved in 500ml of H_2O solution and depression in freezing point of solution is 0.5°C

Find percentage dissociation .

$\left(K_f\right)_{H_2O} = 1.86\ \text{k kg mole}^{-1}$

Ans. **(7.5)**

Sol. $\Delta T_f = i \times K_f \times m$

$$0.5 = (1+\alpha) \times 1.86 \times \frac{9.45 \times 1000}{94.5 \times 500}$$

$\Rightarrow (1+\alpha) = 1.075$

$\Rightarrow \alpha = 0.075$

$\Rightarrow \alpha = 7.5\%$

27. What is the coordination number in Body centered cubic (BCC) arrangement of identical particles

Ans. **8**

Sol. Theory

28. Among the following compounds how many are amphoteric in nature

$Be(OH)_2$, BeO, $Ba(OH)_2$, $Sr(OH)_2$

Ans. **2**

Sol. $Be(OH)_2$, BeO

29. 4.5 gm of solute having molar mass of 90 gm/mol is dissolved in water to make 250 ml solution. Calculate molarity of the solution

Ans. **0.2**

Sol. $M = \frac{n}{V} = \frac{4.5/90}{250/1000} = 0.2$

30. Mass of Li^{3+} is 8.33 times mass of proton Li^{3+} and proton are accelerated through same potential difference. Then ratio of debroglie's wavelength of Li^{3+} to proton is $x \times 10^{-1}$. Find x

Ans. **2**

Sol. $\lambda_{DB}\ \alpha\ \frac{1}{\sqrt{m.K.E.}}$

$$\frac{\lambda_{Li^{3+}}}{\lambda_P} = \sqrt{\frac{m_p \times e_p V}{8.33 m_p \times 3 e_p V}}$$

$$\sqrt{\frac{1}{25}} = \frac{1}{5} = 0.2 = 2 \times 10^{-1}$$

MATHEMATICS

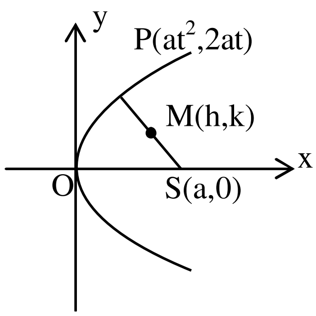
1. The locus of mid-point of the line segment joining focus of parabola $y^2 = 4ax$ to a point moving on it, is a parabola equation of whose directrix is

(1) $y = 0$ (2) $x = 0$ (3) $x = a$ (4) $y = a$

Ans. (2)

Sol. $h = \frac{at^2 + a}{2}$, $k = \frac{2at + 0}{2}$

$\Rightarrow t^2 = \frac{2h - a}{a}$ and $t = \frac{k}{a}$



$\Rightarrow \frac{k^2}{a^2} = \frac{2h - a}{a}$

$\Rightarrow$ Locus of (h, k) is $y^2 = a(2x - a)$

$\Rightarrow y^2 = 2a\left(x - \frac{a}{2}\right)$

Its directrix is $x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$

2. There are 6 Indians 8 foreigners

Find number of committee form with atteast 2 Indians such that number of foreigners is twice the number of Indians.

(1) 1625 (2) 1050 (3) 1400 (4) 575

Ans. (1)

Sol. (2I , 4F) + (3I , 6F) + (4I , 8F)

$= {}^6C_2\,{}^8C_4 + {}^6C_3\,{}^8C_6 + {}^6C_4\,{}^8C_8$

$= 15 \times 70 + 20 \times 28 + 15 \times 1$

$= 1050 + 560 + 15 = 1625$

3. There are two positive number p and q such that $p + q = 2$ and $p^4 + q^4 = 272$. Find the quadratic equation whose roots are p and q.

(1) $x^2 - 2x + 2 = 0$ (2) $x^2 - 2x + 135 = 0$ (3) $x^2 - 2x + 16 = 0$ (4) $x^2 - 2x + 130 = 0$

Ans. (3)

Sol. $(p^2 + q^2)^2 - 2p^2q^2 = 272$

$((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$

$16 - 16pq + 2p^2q^2 = 272$

$(pq)^2 - 8pq - 128 = 0$

$pq = \frac{8 \pm 24}{2} = 16, -8$

$pq = 16$

4. A fair die is thrown n times. The probability of getting an odd number twice is equal to that getting an even number thrice. The probability of getting an odd number, odd number of times is

(1) $\frac{1}{3}$ (2) $\frac{1}{6}$ (3) $\frac{1}{2}$ (4) $\frac{1}{8}$

Ans. (3)

Sol. P(odd no. twice) = P(even no. thrice)

$\Rightarrow {}^nC_2\left(\frac{1}{2}\right)^n = {}^nC_3\left(\frac{1}{2}\right)^n \Rightarrow n = 5$

success is getting an odd number then P(odd successes) = P(1) + P(3) + P(5)

$= {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$

$= \frac{16}{2^5} = \frac{1}{2}$

5. Population of a town at time t is given by the differential equation $\frac{dP(t)}{dt} = (0.5)P(t) - 450$. Also $P(0) = 850$ find the time when population of town becomes zero.

(1) $\ell n9$ (2) $3\ell n4$ (3) $2\ell n18$ (4) $\ell n18$

Ans. (3)

Sol. $\frac{dP(t)}{dt} = \frac{P(t) - 900}{2}$

$\int_0^t \frac{dP(t)}{P(t) - 900} = \int_0^t \frac{dt}{2}$

$\{\ell n\,|P(t) - 900|\}_0^t = \left\{\frac{t}{2}\right\}_0^t$

$\ell n|P(t) - 900| - \ell n|P(0) - 900| = \frac{t}{2}$

$\ell n|P(t) - 900| - \ell n50 = \frac{t}{2}$

Let at $t = t_1$, $P(t) = 0$ hence

$\ell n|P(t) - 900| - \ell n\,50 = \frac{t_1}{2}$

$t_1 = 2\ell n18$

6. Which of following is tautology ?

(1) $A \wedge (A \to B) \to B$ (2) $B \to (A \wedge A \to B)$

(3) $A \wedge (A \vee B)$ (4) $(A \vee B) \wedge A$

Ans. (1)

Sol. $A \wedge (\sim A \vee B) \to B$

$= [(A \wedge \sim A) \vee (A \wedge B)] \to B$

$= (A \wedge B) \to B$

$= \sim A \vee \sim B \vee B$

$= t$

7. The value of $(-{}^{15}C_1 + 2.\,{}^{15}C_2 - 3\,{}^{15}C_3 + ...-15\,{}^{15}C_{15}) + ({}^{14}C_1 + {}^{14}C_3 + ...+ {}^{14}C_{11})$ is

(1) $2^{16} - 14$ (2) $2^{13} - 14$ (3) $2^{13} - 13$ (4) 2^{14}

Ans. (2)

Sol. $S_1 = -{}^{15}C_1 + 2.{}^{15}C_2 - ....-15.{}^{15}C_{15}$

$= \sum_{r=1}^{15}(-1)^r.r.{}^{15}C_r = 15\sum_{r=1}^{15}(-1)^r\ {}^{14}C_{r-1}$

$= 15\,(-{}^{14}C_0 + {}^{14}C_1 -.....-{}^{14}C_{14}) = 15\,(0) = 0$

$S_2 = {}^{14}C_1 + {}^{14}C_3 +..........+ {}^{14}C_{11}$

$= ({}^{14}C_1 + {}^{14}C_3 +.........+ {}^{14}C_{11} + {}^{14}C_{13}) - {}^{14}C_{13}$

$= 2^{13} - 14$

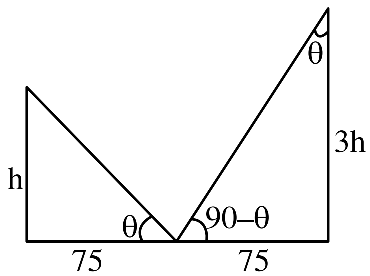
$S_1 + S_2 = 2^{13} - 14$

8. Two towers are 150m distance apart. Height of one tower is thrice the other tower. The angle of elevation of top of tower from midpoint of their feets are complement to each other then the height of smaller tower is

(1) $25\sqrt{3}$ m (2) $\frac{25}{\sqrt{3}}$ m (3) $75\sqrt{3}$ m (4) 25 m

Ans. (1)

Sol.



$\tan\theta = \frac{h}{75} = \frac{75}{3h}$

$\Rightarrow\ h^2 = \frac{(75)^2}{3}$

$h = 25\sqrt{3}$ m

9. Tangent at point $P(t, t^3)$ of curve $y = x^3$ meets the curve again at Q then ordinate of point which divides PQ in 1 : 2 internally , is

(1) 0 (2) $2t^3$ (3) $-2t^3$ (4) 8t

Ans. (3)

Sol. equation of tangent at $P(t,t^3)$

$(y - t^3) = 3t^2 (x - t)$ ....(1)

now solve the above equation with

$y = x^3$ ....(2)

By (1) & (2)

$x^3 - t^3 = 3t^2 (x - t)$

$x^2 + xt + t^2 = 3t^2$

$x^2 + xt - 2t^2 = 0$

$(x - t) (x + 2t) = 0$

$\Rightarrow x = -2t \Rightarrow Q(-2t, -8t^3)$

Ordinate of required point = $\frac{2t^3 + (-8t^3)}{3} = -2t^3$

10. Let $f(x) = \frac{4x^3 - 3x^2}{6} - 2 \sin x + (2x - 1) \cos x$ then f(x)

(1) decreases in $\left[\frac{1}{2}, \infty\right)$ (2) increases in $\left[\frac{1}{2}, \infty\right)$

(3) decreases in $(-\infty, \infty)$ (4) increases in $\left(-\infty, \frac{1}{2}\right]$

Ans. (2)

Sol. $f'(x) = (2x - 1) (x - \sin x)$

$\Rightarrow f'(x) \geq 0$ in $x \in \left[\frac{1}{2}, \infty\right)$

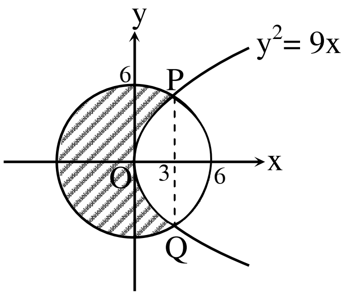
and $f'(x) \leq 0$ in $x \in \left(-\infty, \frac{1}{2}\right]$

11. The area bounded by region inside the circle $x^2 + y^2 = 36$ and outside the parabola $y^2 = 9x$ is

(1) $12\pi + 3\sqrt{3}$ (2) $36\pi + 3\sqrt{3}$ (3) $24\pi - 3\sqrt{3}$ (4) $24\pi - 3\sqrt{3}$

Ans. (3)

Sol. The curves intersect at points $(3, \pm 3\sqrt{3})$



Required area

$$= \pi r^2 - 2\left[\int_0^3 \sqrt{9x}\,dx + \int_3^6 \sqrt{36-x^2}\,dx\right]$$

$$= 36\pi - 12\sqrt{3} - 2\left|\frac{x}{2}\sqrt{36-x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right|_3^6$$

$$= 36p - 12\sqrt{3} - 2\left(9\pi - \left(\frac{9\sqrt{3}}{2} + 3\pi\right)\right) = 24\pi - 3\sqrt{3}$$

12. The equation of plane perpendicular to planes $3x + y - 2z + 1 = 0$ and $2x - 5y - z + 3 = 0$ such that it passes through point $(1, 2, -3)$

(1) $11x + y + 17z + 38 = 0$ (2) $11x - y - 17z + 40 = 0$

(3) $11x + y - 17z + 36 = 0$ (4) $x + 11y + 17z + 3 = 0$

Ans. (1)

Sol. Normal vector of required plane is $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} - 17\hat{k}$

$\therefore +11(x - 1) + (y - 2) + 17(z + 3) = 0$

$11x + y + 17z + 38 = 0$

13. If $f : R \to R$ is a function defined by $f(x) = [x - 1] \cos\left(\frac{2x-1}{2}\right)\pi$, where[x] denotes the greatest integer function, then f is :

(1) continuous for every real x.

(2) discontinuous only at $x = 1$.

(3) discontinuous only at non-zero integral values of x.

(4) continuous only at $x = 1$.

Ans. (1)

Sol. Doubtful points are $x = n, n \in I$

$$\text{L.H.L} = \lim_{x \to n^-} [x-1] \cos\left(\frac{2x-1}{2}\right)\pi = (n-2)\cos\left(\frac{2n-1}{2}\right)\pi = 0$$

$$\text{R.H.L.} = \lim_{x \to n^+} [x-1] \cos\left(\frac{2x-1}{2}\right)\pi = (n-1)\cos\left(\frac{2n-1}{2}\right)\pi = 0$$

$f(n) = 0$

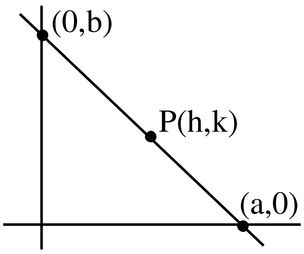
Hence continuous

14. A point is moving on the line such that the AM of reciprocal of interscepts on axis is $\frac{1}{4}$. There are 3 stones whose position are (2, 2) (4, 4) and (1, 1). Find the stone which satisfies the line

(1) (2, 2) (2) (4, 4) (3) (1, 1) (4) All of above

Ans. (1)

Sol.



$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{h}{a} + \frac{k}{b} = 1 \qquad \text{........ (i)}$$

$$\frac{\frac{1}{a}+\frac{1}{b}}{2} = \frac{1}{4}$$

$$\therefore \frac{1}{a}+\frac{1}{b}=\frac{1}{2} \qquad \text{........ (ii)}$$

$\therefore$ Line passes through fixed point (2, 2)

(from (1) and (2))

15. If $e^{(\cos^2\theta+\cos^4\theta+....\infty)\ln 2}$ is a root of equation $t^2 - 9t + 8 = 0$ then then value of $\frac{2\sin\theta}{\sin\theta+\sqrt{3}\cos\theta}$ when $0 < \theta < \frac{\pi}{2}$, is

(1) $\frac{1}{2}$ (2) 1 (3) 2 (4) 4

Ans. (1)

Sol. $e^{(\cos^2\theta+\cos^4\theta+....\infty)\ln 2} = 2^{\cos^2\theta+\cos^4\theta+....\infty}$

$= 2^{\cot^2\theta}$

$t^2 - 9t + 8 = 0 \Rightarrow t = 1, 8$

$\Rightarrow 2^{\cot^2\theta} = 1, 8 \Rightarrow \cot^2\theta = 0, 3$

$0 < \theta < \frac{\pi}{2} \Rightarrow \cot\theta = \sqrt{3}$

$\Rightarrow \frac{2\sin\theta}{\sin\theta + \sqrt{3}\sin\theta} = \frac{2}{1+\sqrt{3}\cot\theta} = \frac{2}{4} = \frac{1}{2}$

16. If $I = \int \frac{\cos\theta - \sin\theta}{\sqrt{8-\sin 2\theta}}\, d\theta = a\sin^{-1}\left(\frac{\sin\theta + \cos\theta}{b}\right) + C$

then ordered pair (a, b) is

(1) (1, 3) (2) (3, 1) (3) (1, 1) (4) (–1, 3)

Ans. (1)

Sol. put $\sin\theta + \cos\theta = t \Rightarrow 1 + \sin 2\theta = t^2$

$\Rightarrow (\cos\theta - \sin\theta)\, d\theta = dt$

$\therefore I = \int \frac{dt}{\sqrt{8-(t^2-1)}} = \int \frac{dt}{\sqrt{9-t^2}} = \sin^{-1}\left(\frac{t}{3}\right) + C = \sin^{-1}\left(\frac{\sin\theta+\cos\theta}{3}\right) + C$

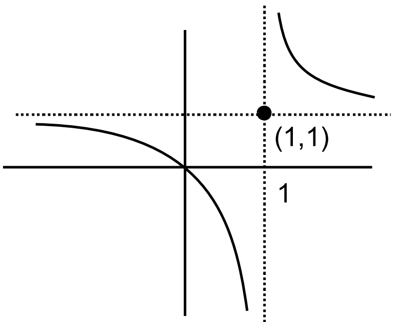
$\Rightarrow a = 1$ and $b = 3$

17. Such that $f: R \to R$, $f(x) = 2x - 1$, $g(x) = \frac{x - \frac{1}{2}}{x-1}$, then f(g(x)) is

(1) one-one, onto (2) many-one, onto (3) one-one, into (4) many-one, into

Ans. (3)

Sol.



$f(g(x)) = 2g(x) - 1$

$= 2\frac{\left(x-\frac{1}{2}\right)}{x-1} = \frac{x}{x-1}$

$f(g(x)) = 1 + \frac{1}{x-1}$

one-one, into

18. The distance of the point P(1,1,9) from the point of intersection of plane x + y + z = 17 and line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$

(1) $\sqrt{38}$ (2) $\sqrt{39}$ (3) 6 (4) 7

Ans. (1)

Sol. $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=\lambda$

$\Rightarrow x=\lambda+3\ ,\ y=2\lambda+4,\ z=2\lambda+5$

Which lies on given plane hence

$\Rightarrow \lambda+3+2\lambda+4+2\lambda+5=17$

$\Rightarrow \lambda=\frac{5}{5}=1$

Hence , point of intersection is Q (4,6,7)

$\therefore$ Required distance = PQ

$= \sqrt{9+25+4}$

$= \sqrt{38}$

19. The value of $\lim_{x\to0^+}\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3}$

(1) $\frac{1}{15}$ (2) $\frac{2}{3}$ (3) 3 (4) 2

Ans. (2)

Sol. $\lim_{x\to0^+}\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3}=\lim_{x\to0^+}\frac{(\sin|x|)2x}{3x^2}=\lim_{x\to0^+}\left(\frac{\sin x}{x}\right)\times\frac{2}{3}=\frac{2}{3}$

20. The values of k and m such that system of equations 3x + 2y – k z = 10, x – 2y + 3z = 3, x + 2y – 3z = 5m are inconsistent.

(1) $k=3$ and $m\neq\frac{7}{10}$ (2) $k=3$ and $m=\frac{7}{10}$

(3) $k\neq3$ and $m=\frac{7}{10}$ (4) $k=2$ and $m\neq\frac{7}{10}$

Ans. (1)

Sol. $\Delta = \begin{vmatrix} 3 & 2 & -k \\ 1 & -2 & 3 \\ 1 & 2 & -3 \end{vmatrix} = 0 \Rightarrow k = 3$

$$\Delta_x = \begin{vmatrix} 10 & 2 & -3 \\ 3 & -2 & 3 \\ 5m & 2 & -3 \end{vmatrix} = 0$$

$$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 1 & 3 & 3 \\ 1 & 5m & -3 \end{vmatrix} = 6(7-10m)$$

$$\Delta_z = \begin{vmatrix} 3 & 2 & 10 \\ 1 & -2 & 3 \\ 1 & 2 & 5m \end{vmatrix} = 4(7-10m)$$

Hence , k = 3 and $m \neq \frac{7}{10}$

21. $\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r^2+r}\right)\right) =$

Ans. 01.00

Sol. $\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\left[\tan^{-1}(r+1)-\tan^{-1}(r)\right]\right)$

$= \tan\left(\lim_{n\to\infty}\left(\tan^{-1}(n+1)-\frac{\pi}{4}\right)\right)$

$= \tan\left(\frac{\pi}{4}\right) = 1$

22. Of the three independent events B_1, B_2 and B_3, the probability that only B_1 occurs is α,only B_2 occurs is β and only B_3 occurs is γ. Let the probability p that none of events B_1, B_2 or B_3 occurs satisfy the equations $(\alpha - 2\beta)\ p = \alpha\beta$ and $(\beta - 3\gamma)\ p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then $\dfrac{\text{Probability of occurrence of } B_1}{\text{Probability of occurrence of } B_3} =$

Ans. 6

Sol. Let x, y, z be probability of B_1, B_2, B_3 respectively

$\Rightarrow\ x(1-y)(1-z) = \alpha \qquad \Rightarrow\ y(1-x)(1-z) = \beta$

$\Rightarrow\ z(1-x)(1-y) = \gamma \qquad \Rightarrow\ (1-x)(1-y)(1-z) = P$

Putting in the given relation we get x = 2y and y = 3z $\Rightarrow$ x = 6z $\Rightarrow$ $\frac{x}{z} = 6$

23. $\vec{c}$ is coplanar with $\vec{a}=-\hat{i}+\hat{j}+\hat{k}$ & $\vec{b}=2\hat{i}+\hat{k}$, $\vec{a}.\vec{c}=7$ & $\vec{c}\perp\vec{b}$. then the value of $2\left|\vec{a}+\vec{b}+\vec{c}\right|^2$ is .

Ans. 75.00

Sol. $\vec{c}=\lambda\left(\vec{b}\times\left(\vec{a}\times\vec{b}\right)\right)$

$=\lambda\left(\left(\vec{b}.\vec{b}\right)\vec{b}-\left(\vec{b}.\vec{a}\right)\vec{b}\right)$

$=\lambda\left(5\left(-\hat{i}+\hat{j}+\hat{k}\right)+2\hat{i}+\hat{k}\right)$

$=\lambda\left(-3\hat{i}+5\hat{j}+6\hat{k}\right)$

$\vec{c}.\vec{a}=7\Rightarrow 3\lambda+5\lambda+6\lambda=7$

$\lambda=\frac{1}{2}$

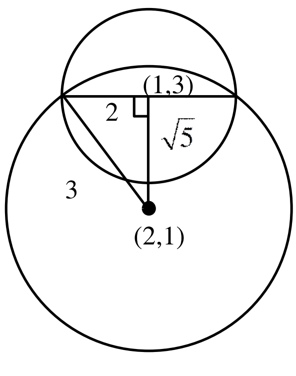
$\therefore 2\left|\left(\frac{-3}{2}-1+2\right)\hat{i}+\left(\frac{5}{2}+1\right)\hat{j}+(3+1+1)\hat{k}\right|^2$

$=2\left(\frac{1}{4}+\frac{49}{4}+25\right)$ = 25 + 50 = 75

24. One of the diameter of circle $C_1 : x^2 + y^2 - 2x - 6y + 6 = 0$ is chord of circle C_2 with centre (2, 1) then radius of C_2 is

Ans. **3**

Sol.



distance between (1, 3) and (2, 1) is $\sqrt{5}$

$\therefore \left(\sqrt{5}\right)^2 + (2)^2 = r^2$

$\Rightarrow r = 3$

25. Let $P=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix such that PQ = k I, where $k \in R$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and det (Q) = $\frac{k^2}{2}$, then value of $k^2 + \alpha^2$ is equal to

Ans. 17

Sol. As $PQ = kI \Rightarrow Q = kP^{-1}I$

now $Q = \frac{k}{|P|}(\text{adj}P)\, I \Rightarrow Q = \frac{k}{(20+12\alpha)}\begin{bmatrix} - & - & - \\ - & - & (-3\alpha-4) \\ - & - & - \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\because q_{23} = \frac{-k}{8} \Rightarrow \frac{k}{(20+12\alpha)}(-3\alpha-4) = \frac{-k}{8} \Rightarrow 2(3\alpha+4) = 5+3\alpha$

$3\alpha = -3 \Rightarrow \alpha = -1$

also $|Q| = \frac{k^3|I|}{|P|} \Rightarrow \frac{k^2}{2} = \frac{k^3}{(20+12\alpha)}$

$(20+12\alpha) = 2k \Rightarrow 8 = 2k \Rightarrow k = 4$

26. How many 3 × 3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of $M^T M$ is 7 ?

Ans. 540

Sol. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case I : Seven (1's) and two (0's)

$^9C_2 = 36$

Case- II : One (2) and three (1's) and five (0's)

$\frac{9!}{5!3!} = 504$

$\therefore$ Total = 540

27. $z + \alpha|z-1| + 2i = 0$; $z \in C$ & $\alpha \in R$, then the value of $4[(\alpha_{max})^2 + (\alpha_{min})^2]$ is

Ans. 10

Sol. $x + iy + \alpha\sqrt{(x-1)^2+y^2} + 2i = 0$

$\therefore$ $y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2+y^2} = 0$

$y = -2$ & $x^2 = \alpha^2(x^2 - 2x + 1 + 4)$

$\alpha^2 = \frac{x^2}{x^2-2x+5}$

$\alpha^2 \in \left[0, \frac{5}{4}\right]$

$\therefore$ $\alpha^2 \in \left[0, \frac{5}{4}\right]$

$\therefore$ $\alpha \in \left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$

then $4[(\alpha_{max})^2 + (\alpha_{min})^2] = 4\left[\frac{5}{4}+\frac{5}{4}\right] = 10$

28. Let A = {x : x is 3 digit number}

B = {x : x = 9K + 2, k $\in$ I}

C : {x : x = 9K + ℓ, k $\in$ I, $\ell \in$ I, $0 < \ell < 9$}

If sum of elements in $A \cap (B \cup C)$ is 274×400 then ℓ is

Ans. 5.00

Sol. 3 digit number of the form 9K + 2 are {101,109,...............,992}

$\Rightarrow$ Sum equal to $\frac{100}{2}$ (1093)

Similarly sum of 3 digit number of the form 9K + 5 is $\frac{100}{2}$ (1099)

$$\frac{100}{2}(1093) + \frac{100}{2}(1099) = 100 \times (1096)$$

$$= 400 \times 274$$

$$\Rightarrow \ell = 5$$

29. The least value of α such that $\frac{4}{\sin x} + \frac{1}{1-\sin x} = \alpha$ has at least one solution in $x \in \left(0, \frac{\pi}{2}\right)$

Ans. 9.00

Sol. Let $f(x) = \frac{4}{\sin x} + \frac{1}{1-\sin x}$

$$y = \frac{4 - 3\sin x}{\sin x(1-\sin x)}$$

Let sin x = t when t $\in$ (0,1)

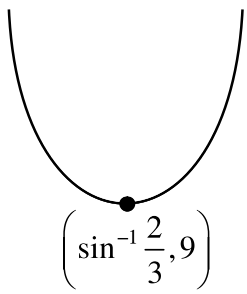
$$y = \frac{4-3t}{t-t^2}$$

$$\frac{dy}{dt} = \frac{-3(t-t^2)-(1-2t)(4-3t)}{(t-t^2)^2} = 0$$

$\Rightarrow 3t^2 - 3t - (4 - 11t + 6t^2) = 0$

$\Rightarrow 3t^2 - 8t + 4 = 0$

$\Rightarrow 3t^2 - 6t - 2t + 4 = 0$



$\Rightarrow t = \frac{2}{3}$

$\Rightarrow \alpha \geq 9$

least α is equal to 9

30. $\int_{-a}^{a} |x|+|x-2| = 22$, a > 2 then the value of $\int_{-a}^{a} x+[x]$ is

(where [.] represent greatest integer function)

Ans. –3

Sol. $\int_{-a}^{0} (-2x+2)\ dx + \int_{0}^{2} (x+2-x)\ dx + \int_{2}^{a} (2x-2)\ dx = 22$

$x^2-2x\Big|_0^{-a} + 2x\Big|_0^2 + x^2-2x\Big|_2^a = 22$

$a^2 + 2a + 4 + a^2 - 2a - (4 - 4) = 22$

$2a^2 = 18 \Rightarrow a = 3$

$\int_{-3}^{3} (x+[x])\ dx = -3 - 2 - 1 + 1 + 2 = -3$